量子計算の場の量子論への応用について

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(本多正純)

“Application of Quantum Computation to Quantum Field Theory (QFT)” ??

This talk = applications in two directions

1. practical

use quantum computer to simulate QFT

2. conceptual (?)

possible relations between gauge theory & quantum error correction

Plan

1. Practical applications

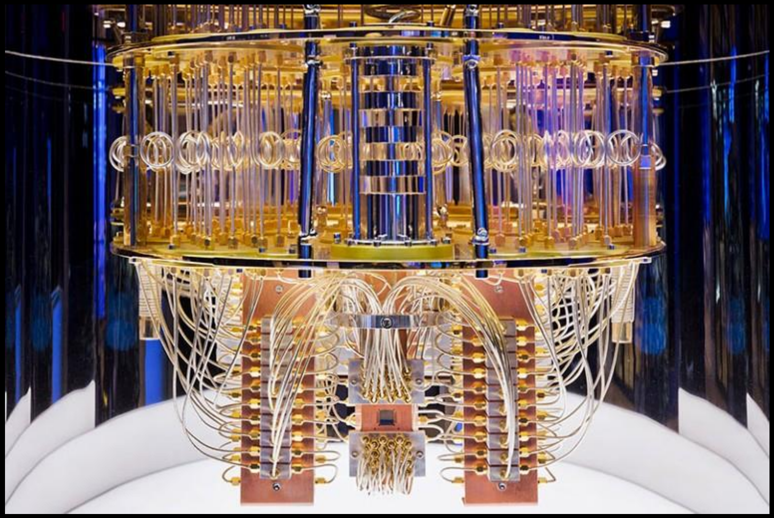
- Introduction
- QFT as qubits
- Schwinger model [Chakraborty-MH-Kikuchi-Izubuchi-Tomiya ’20, MH-Itou-Kikuchi-Nagano-Okuda ’21, MH-Itou-Kikuchi-Tanizaki ’21, MH-Itou-Tanizaki ‘22]
- Recent attempts

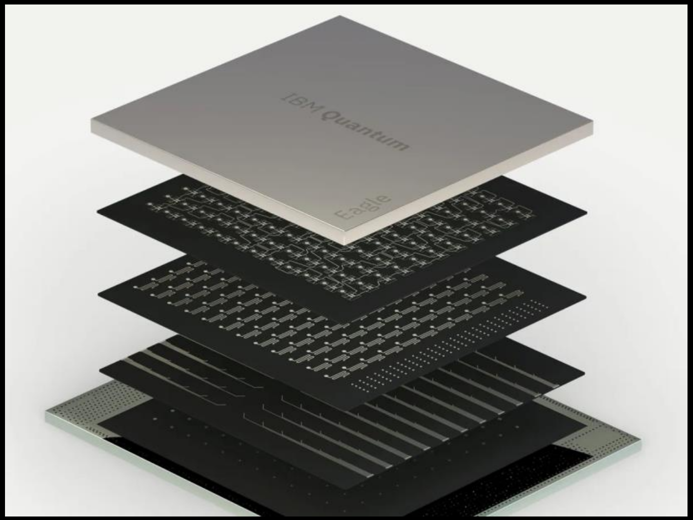
2. Conceptual application

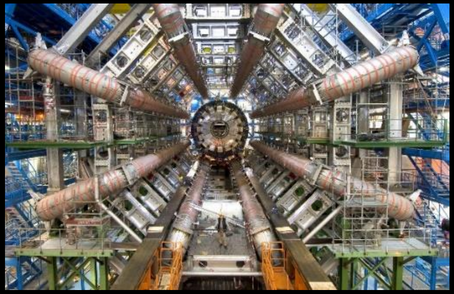
- Introduction
- Lightning review of QEC (quantum error correction)
- QEC & Gauge theory

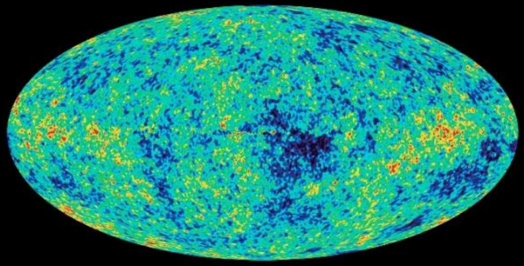
3. Outlook

Vision of practical applications









etc…

This talk:

Application of Quantum Computation to Quantum Field Theory (QFT)

▪ Generic motivation:

simply would like to use powerful computers?

▪ Specific motivation:

Quantum computation is suitable for operator formalism

$\longrightarrow$ Liberation from infamous sign problem in Monte Carlo?

Cost of operator formalism

We have to play with huge vector space

since QFT typically has ∞-dim. Hilbert space

regularization needed!

Technically, computers have to

memorize huge vector & multiply huge matrices

Quantum computers do this job?

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“Regularization” of Hilbert space

Hilbert space of QFT is typically ∞ dimensional

$\longrightarrow$ Make it finite dimensional!

- Fermion is easiest (up to doubling problem)
  — Putting on spatial lattice, Hilbert sp. is finite dimensional
- scalar
  — Hilbert sp. at each site is ∞ dimensional
  (need truncation or additional regularization)
- gauge field (w/ kinetic term)
  — no physical d.o.f. in 0+1D/1+1D (w/ open bdy. condition)
  — ∞ dimensional Hilbert sp. in higher dimensions

(1+1)d free Dirac fermion (continuum)

Lagrangian:

$$\mathcal{L} = \int dx \left[i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi \right] \qquad \{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu}$$

$$\Bigg\Downarrow \qquad \frac{\partial \mathcal{L}}{\partial(\partial_t \psi)} = \bar{\psi}$$

Hamiltonian:

$$H = \int dx \left[-i\bar{\psi}\gamma^{1}\partial_{1}\psi + m\bar{\psi}\psi \right]$$

$$\{\psi(x), \bar{\psi}(y)\} = \delta(x - y)$$

(1+1)d free Dirac fermion (lattice)

Continuum:

$$\psi(x) = \begin{pmatrix} \psi_u(x) \\ \psi_d(x) \end{pmatrix} \qquad \gamma^0 = \sigma_3, \quad \gamma^1 = i\sigma_2$$

$$H = \int dx \left[-i\bar{\psi}\gamma^1\partial_1\psi + m\bar{\psi}\psi \right]$$

$$= \int dx \left[-i(\psi_u^\dagger \partial_1 \psi_d + \psi_d^\dagger \partial_1 \psi_u) + m(\psi_u^\dagger \psi_u - \psi_d^\dagger \psi_d) \right]$$

$$\Downarrow$$

Lattice (w/ N sites and spacing a):

"Staggered fermion" [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix} \begin{matrix} \rightarrow \text{odd site} \\ \rightarrow \text{even site} \end{matrix}$$

(1+1)d free Dirac fermion (lattice)

Continuum:

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$$= \int dx\left[-i(\psi_u^\dagger\partial_1\psi_d + \psi_d^\dagger\partial_1\psi_u) + m(\psi_u^\dagger\psi_u - \psi_d^\dagger\psi_d)\right]$$

$$\Downarrow$$

Lattice (w/ N sites and spacing a):

"*Staggered fermion*" [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix}\psi_u\\ \psi_d\end{pmatrix}$$

ψ_u → odd site

ψ_d → even site

$$H = -\frac{i}{2a}\sum_{n=1}^{N-1}\left(\chi_n^\dagger\chi_{n+1} - \chi_{n+1}^\dagger\chi_n\right) + m\sum_{n=1}^{N}(-1)^n\chi_n^\dagger\chi_n$$

$$\{\chi_m, \chi_n^\dagger\} = \delta_{\mathrm{mn}}, \ \{\chi_m, \chi_n\} = 0$$

Jordan-Wigner transformation

$$\{\chi_m, \chi_n^\dagger\} = \delta_{\mathrm{mn}}, \ \{\chi_m, \chi_n\} = 0$$

This is satisfied by the operator: [Jordan-Wigner'28]

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2} \left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i \right)$$

(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

Jordan-Wigner transformation

$$\{\chi_m, \chi_n^\dagger\} = \delta_{\mathrm{mn}}, \ \{\chi_m, \chi_n\} = 0$$

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(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

Then the system is mapped to the spin system:

$$\boxed{\hat{H} = \frac{w}{2}\sum_{n=1}^{N-1}\left(X_n X_{n+1} + Y_n Y_{n+1}\right) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n}$$

Now we can apply quantum algorithms to QFT!

Scalar field theory (continuum)

Lagrangian:

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)$$

$$\Downarrow \qquad \Pi(\mathbf{x}) = \frac{\partial\mathcal{L}}{\partial(\partial_t\phi)} = \partial_t\phi$$

Hamiltonian:

$$\mathcal{H}(\boldsymbol{x}) = \frac{1}{2}\Pi^2 + \frac{1}{2}(\partial_i\phi)^2 + V(\phi)$$

$$[\phi(\boldsymbol{x}), \Pi(\boldsymbol{y})] = i\delta^{(d)}(\boldsymbol{x} - \boldsymbol{y})$$

Scalar field theory (lattice)

Continuum Hamiltonian:

$$H = \int d^d\boldsymbol{x} \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\partial_i\phi)^2 + V(\phi)\right]$$

$$\int d^dx \to a^d \sum_n ,$$

$$\partial_\mu\phi(x) \to \Delta_\mu\phi(x_n) \equiv \frac{\phi(x_n+ae_\mu)-\phi(x_n)}{a}$$

Lattice Hamiltonian (simplest):

$$H = a^d \sum_n \left[\frac{1}{2}\Pi_n^2 + \frac{1}{2}\sum_i(\Delta_i\phi_n)^2 + V(\phi_n)\right]$$

$$[\phi(\boldsymbol{x_m}), \Pi(\boldsymbol{x_n})] = i\delta_{\boldsymbol{m},\boldsymbol{n}}$$

technically the same as multi-particle QM

Regularization for single particle QM

$$\hat{H} = \frac{1}{2}\hat{p}^2 + \frac{\omega^2}{2}\hat{x}^2 + V_{\text{int}}(\hat{x})$$

Most naïve approach = truncation in harmonic osc. basis:

$$\hat{a} = \sqrt{\frac{\omega}{2}}\,\hat{x} + \frac{i}{\sqrt{2\omega}}\,\hat{p} \;\; = \;\; \sum_{n=0}^{\infty} \sqrt{n+1}\,|n\rangle\langle n+1|$$

$$\xRightarrow[\textit{regularize!}]{} \;\; \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

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$$\xrightarrow{\textit{regularize!}}\; \sum_{n=0}^{\Lambda-2}\sqrt{n+1}\,|n\rangle\langle n+1|$$

Then replace $\hat{p}$ & $\hat{x}$ by

$$\hat{x}\Big|_{\text{regularized}} \equiv \frac{1}{\sqrt{2\omega}}(\hat{a} + \hat{a}^\dagger)\Big|_{\text{regularized}}$$

$$\hat{p}\Big|_{\text{regularized}} \equiv \frac{1}{i}\sqrt{\frac{\omega}{2}}\,(\hat{a} - \hat{a}^\dagger)\Big|_{\text{regularized}}$$

Regularization for single particle QM (Cont'd)

$$\hat{a}\Big|_{\text{regularized}} = \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

We can rewrite the Fock basis in terms of qubits:

$$|n\rangle = |b_{K-1}\rangle|b_{K-2}\rangle \cdots |b_0\rangle \qquad K \equiv \log_2 \Lambda$$

$$n = \mathrm{b}_{\mathrm{K}-1}2^{\mathrm{K}-1} + \mathrm{b}_{\mathrm{K}-2}2^{\mathrm{K}-2} + \cdots + b_0 2^0$$ (binary representation)

Regularization for single particle QM (Cont'd)

$$\hat{a}\Big|_{\text{regularized}} = \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

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$$n = \mathrm{b}_{\mathrm{K}-1}2^{\mathrm{K}-1} + \mathrm{b}_{\mathrm{K}-2}2^{\mathrm{K}-2} + \cdots + b_0 2^0$$ (binary representation)

Then,

$$|n\rangle\langle n+1| = \otimes_{\ell=0}^{K-1} \underbrace{(|b'_\ell\rangle\langle b_\ell|)}_{\text{either one of}}$$

$$|0\rangle\langle 0| = \frac{1_2 - \sigma_z}{2}, \qquad |1\rangle\langle 1| = \frac{1_2 + \sigma_z}{2},$$
$$|0\rangle\langle 1| = \frac{\sigma_x + i\sigma_y}{2}, \qquad |1\rangle\langle 0| = \frac{\sigma_x - i\sigma_y}{2}$$

Pure Maxwell theory (continuum)

Lagrangian:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad (F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$$

$$\Downarrow$$

temporal gauge $A_0 = 0$

$$E^i = \frac{\partial \mathcal{L}}{\partial \dot{A}_i} = \dot{A}^i$$

Hamiltonian:

$$\mathcal{H} = \frac{1}{2} E_i^2 + \frac{1}{2} B_i^2$$

$$[A_i(\boldsymbol{x}), E_j(\boldsymbol{y})] = i\delta_{ij}\delta^{(d)}(\boldsymbol{x} - \boldsymbol{y})$$

Gauss law:

$$\partial_i E^i = 0$$

Pure Maxwell theory (lattice)

Continuum:

$$\mathcal{H} = \frac{1}{2}E_i^2 + \frac{1}{2}B_i^2 \qquad \partial_i E^i = 0$$

$$\Downarrow$$

Lattice:

$$\mathcal{H} = \frac{a^d}{2}\sum_{\boldsymbol{n},i} L_{\boldsymbol{n},i}^2 + \mathrm{Re}\sum_{\text{plaquette}}\sum_{i<j}\prod_{P\in\text{plaquette}} U_P$$

$$[U_{\boldsymbol{m},i}, L_{\boldsymbol{n},j}] = i\delta_{ij}\delta_{\boldsymbol{m},\boldsymbol{n}}$$

Gauss law:

$$\sum_i (L_{\boldsymbol{n}+\boldsymbol{e}_i,i} - L_{\boldsymbol{n},i}) = 0$$

Ex. (1+1)d pure Maxwell theory w/ θ

Continuum:

$$\mathcal{L} = \frac{1}{2g^2} F_{01}^2 + \frac{\theta}{2\pi} F_{01} \xrightarrow{\Pi = \frac{1}{g^2}\dot{A} + \frac{\theta}{2\pi}} \mathcal{H} = \frac{1}{2}\left(\Pi - \frac{\theta}{2\pi}\right)^2$$

Lattice:

$$H = \frac{g^2 a}{2} \sum_n \left(L_n + \frac{\theta}{2\pi}\right)^2 \qquad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$

Gauss law:

$$L_{n+1} - L_n = 0$$

Ex. (1+1)d pure Maxwell theory w/ θ

Continuum:

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Lattice:

$$H = \frac{g^2 a}{2} \sum_n \left(L_n + \frac{\theta}{2\pi}\right)^2 \qquad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$

Gauss law:

$$L_{n+1} - L_n = 0$$

- open b.c.

$$L_n = L_{n-1} = L_{n-2} = \cdots = L_1 = (b.c.)$$

- p.b.c.

$$L_n = L_{n-1} = \cdots = L_1 = \cdots = L_{n+1} = L_n$$

one d.o.f. remains

Short summary

(repeated)

Hilbert space of QFT is typically ∞ dimensional

⟶ Make it finite dimensional!

- Fermion is easiest (up to doubling problem)

  — Putting on spatial lattice, Hilbert sp. is finite dimensional

- scalar

  — Hilbert sp. at each site is ∞ dimensional

  (need truncation or additional regularization)

- gauge field (w/ kinetic term)

  — no physical d.o.f. in 0+1D/1+1D (w/ open bdy. condition)

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Charge-q Schwinger model

Continuum:

$$L = \frac{1}{2g^2} F_{01}^2 + \frac{\theta_0}{2\pi} F_{01} + \overline{\psi}\, \mathrm{i}\, \gamma^\mu (\partial_\mu + \mathrm{i}\, q\, A_\mu)\psi - m\, \overline{\psi}\psi$$

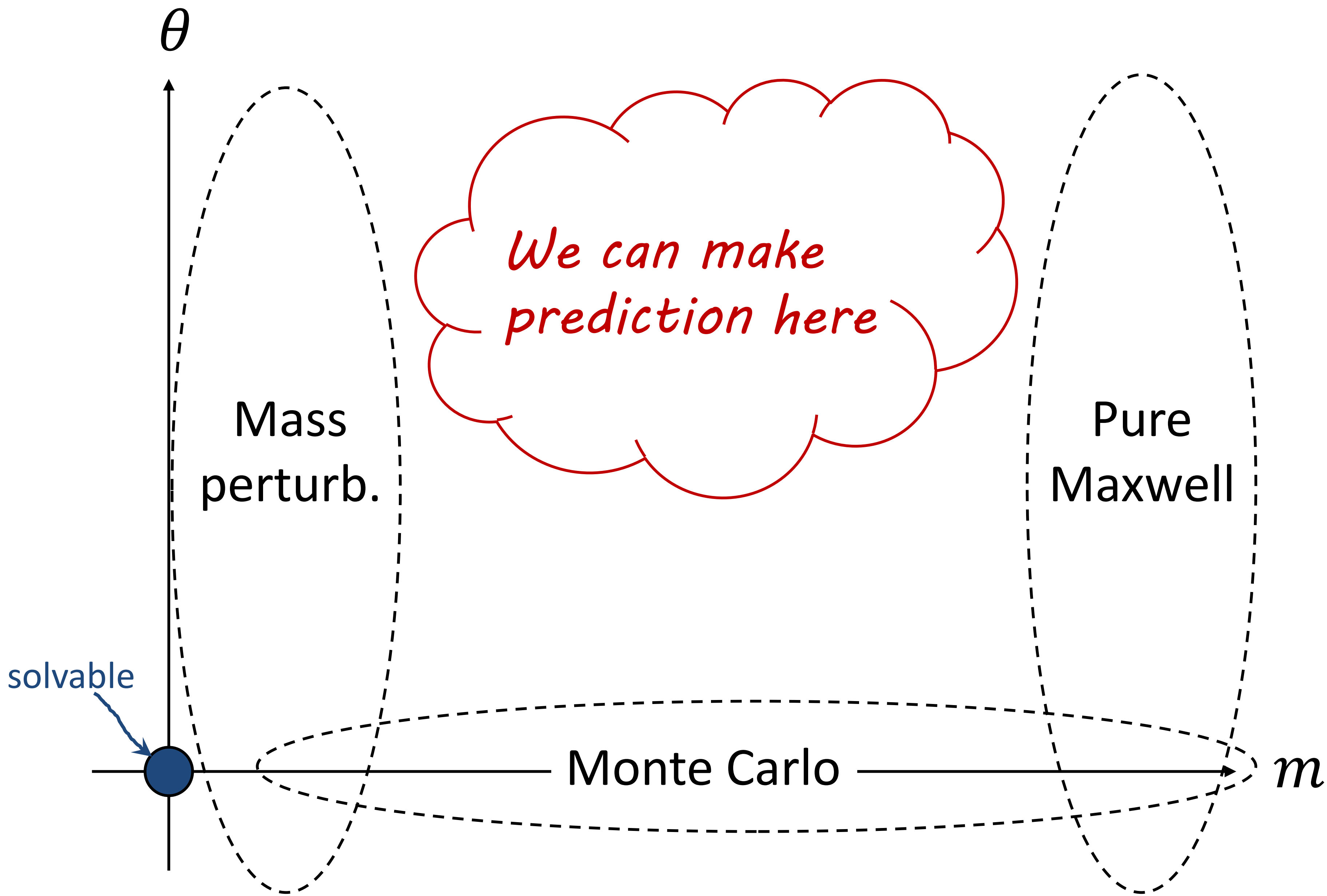
Taking temporal gauge $A_0 = 0$, (Π: conjugate momentum of A_1)

$$H(x) = \frac{g^2}{2}\left(\Pi - \frac{\theta_0}{2\pi}\right)^2 - \bar{\psi}\, \mathrm{i}\, \gamma^1 (\partial_1 + \mathrm{i}\, q A_1)\psi + m\bar{\psi}\psi,$$

Physical states are constrained by Gauss law:

$$0 = -\partial_1 \Pi - qg\bar{\psi}\gamma^0\psi$$

Map of accessibility/difficulty



Lattice theory w/ staggered fermion

Hamiltonian:

$$H = J \sum_{n=0}^{N-2} \left(L_n + \frac{\theta_0}{2\pi} \right)^2 - \mathrm{i}w \sum_{n=0}^{N-2} \left[\chi_n^\dagger (U_n)^q \chi_{n+1} - \mathrm{h.c.} \right] + m \sum_{n=0}^{N-1} (-1)^n \chi_n^\dagger \chi_n$$

$$\left(w = \frac{1}{2a}, J = \frac{g^2 a}{2} \right)$$

Commutation relation:

$$[L_n, U_m] = U_m \delta_{nm}, \quad \{\chi_n, \chi_m^\dagger\} = \delta_{nm}$$

Gauss law:

$$L_n - L_{n-1} = q \left[\chi_n^\dagger \chi_n - \frac{1 - (-1)^n}{2} \right]$$

Eliminate gauge d.o.f.

1. Take open b.c. & solve Gauss law:

$$L_n = L_{-1} + q\sum_{j=1}^{n}\left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right) \qquad \text{w/ } L_{-1} = 0$$

2. Take the gauge $U_n = 1$

Then,

$$H = -\mathrm{i}w\sum_{n=1}^{N-1}\left[\chi_n^\dagger \chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=1}^{N}(-1)^n \chi_n^\dagger \chi_n$$
$$+J\sum_{n=1}^{N}\left[\frac{\theta_0}{2\pi} + q\sum_{j=1}^{n}\left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right)\right]^2.$$

This acts on finite dimensional Hilbert space

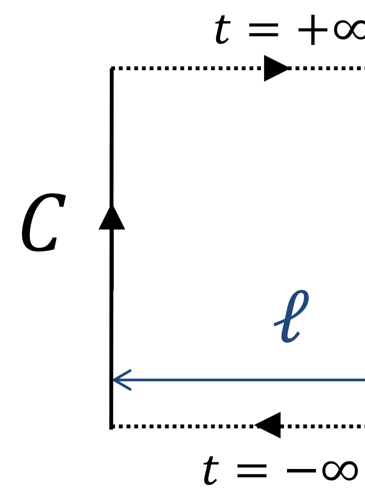
Insertion of the probe charges

① Introduce the probe charges $\pm q_p$:

$$e^{iq_p \int_C A}$$

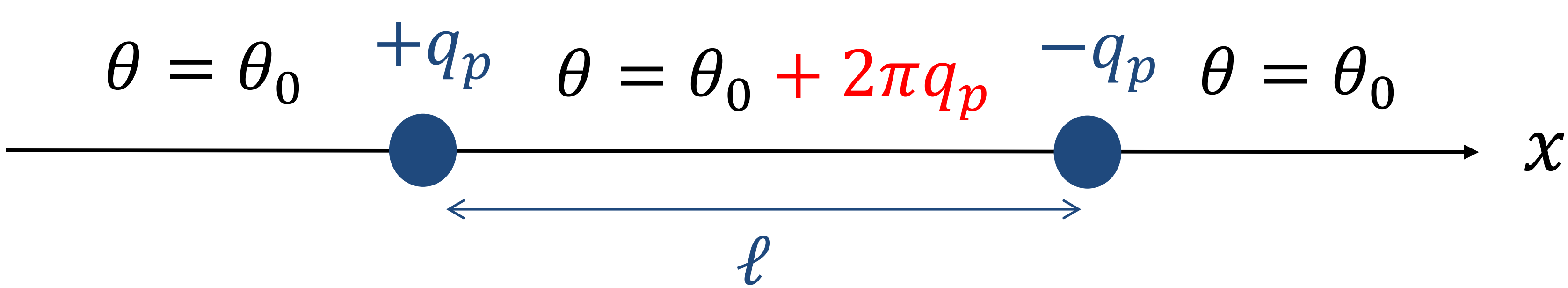
$$||$$

$$e^{iq_p \int_{S, \partial S = C} F}$$



local θ-term w/ $\theta = 2\pi q_p$!!

② Include it to the action & switch to Hamilton formalism



③ Compute the ground state energy (in the presence of the probes)

Going to spin system

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \ \{\chi_n, \chi_m\} = 0$$

This is satisfied by the operator:

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i\right)$$

(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

Now the system is purely a spin system:

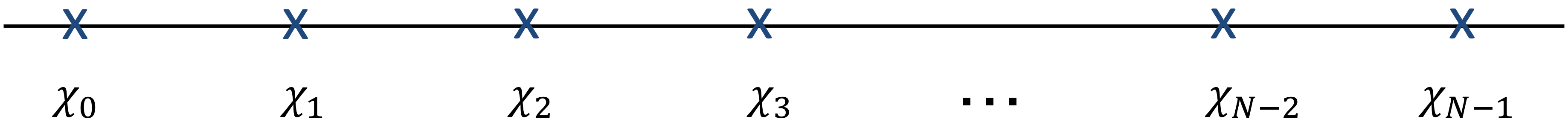
$$H = -\mathrm{i}w\sum_{n=1}^{N-1}\left[\chi_n^\dagger \chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=1}^{N}(-1)^n \chi_n^\dagger \chi_n \ + J\sum_{n=1}^{N}\left[\frac{\vartheta_n}{2\pi} + q\sum_{j=1}^{n}\left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right)\right]^2$$

$$\Downarrow$$

$$H = J\sum_{n=0}^{N-2}\left[q\sum_{i=0}^{n}\frac{Z_i + (-1)^i}{2} + \frac{\vartheta_n}{2\pi}\right]^2 + \frac{w}{2}\sum_{n=0}^{N-2}\left[X_n X_{n+1} + Y_n Y_{n+1}\right] + \frac{m}{2}\sum_{n=0}^{N-1}(-1)^n Z_n$$

Qubit description of the Schwinger model !!

Even N or odd N?



Staggered fermion: $\dfrac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} \begin{array}{l} \longrightarrow \text{odd site} \\ \longrightarrow \text{even site} \end{array}$

- Usually even N is taken (p.b.c. allows only even N)
- Open b.c. allows both but parity is different: $\chi_n \to i(-1)^n \chi_{N-n-1}$

	n mod 2	$\bar{\psi}\psi \sim \sum_n (-1)^n \chi_n^\dagger \chi_n$	$\bar{\psi}\gamma^5\psi \sim \sum_n (-1)^n (\chi_n^\dagger \chi_{n+1} - \mathrm{h.c.})$
even N	changes	flipped	invariant
odd N	invariant	invariant	flipped

Odd N seems more like the continuum theory?

Constructing ground state

$^{\exists}$various quantum algorithms to construct vacuum:

- adiabatic state preparation
- algorithms based on variational method
- imaginary time evolution

etc…

Here, let's apply

adiabatic state preparation

Adiabatic state preparation

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|\mathrm{vac}_0\rangle$ is known and unique

Step 2:

Step 3:

Adiabatic state preparation

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|\mathrm{vac}_0\rangle$ is known and unique

Step 2: Introduce adiabatic Hamiltonian $H_A(t)$ s.t.

$$\begin{cases} \cdot\ H_A(0) = H_0,\ H_A(T) = H_{\mathrm{target}} \\ \cdot\ \left|\frac{dH_A}{dt}\right| \ll 1 \text{ for } T \gg 1 \end{cases}$$

Step 3:

Adiabatic state preparation

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$$\begin{cases} \cdot\ H_A(0) = H_0,\ H_A(T) = H_{\mathrm{target}} \\ \cdot\ \left|\frac{dH_A}{dt}\right| \ll 1 \text{ for } T \gg 1 \end{cases}$$

Step 3: Use the adiabatic theorem

If $H_A(t)$ has a unique ground state w/ a finite gap for $\forall t$, then the ground state of H_{target} is obtained by

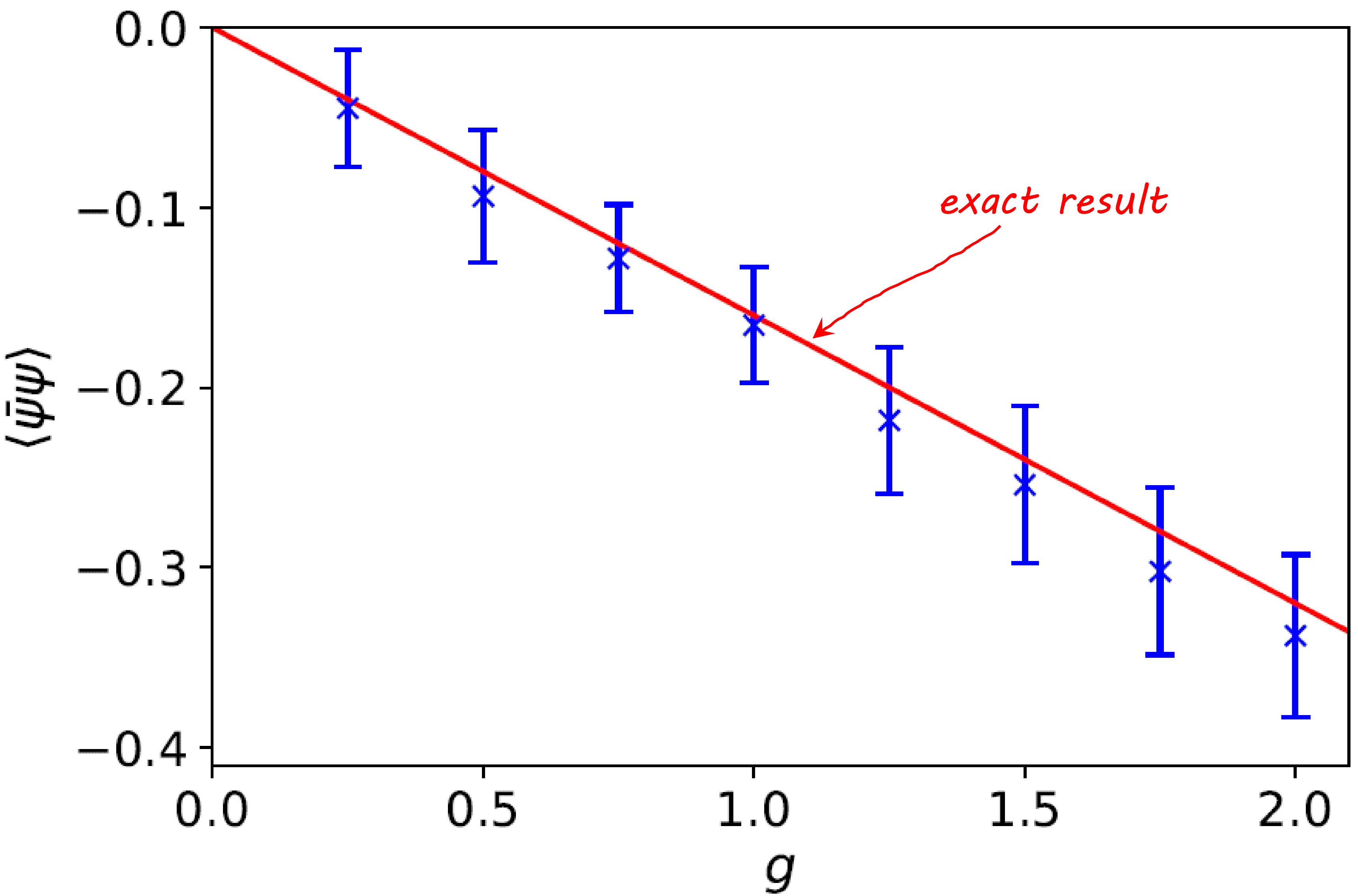
$$\boxed{|\mathrm{vac}\rangle = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\, H_A(t)\right) |\mathrm{vac}_0\rangle}$$

Demo: chiral condensate in massless case

$T = 100, \delta t = 0.1, N_{\max} = 16, 1M$ shots

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

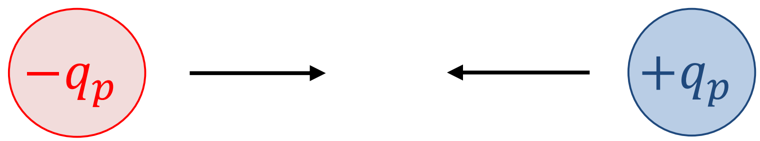
(after continuum limit)



Screening versus Confinement

Let's consider

potential between 2 heavy charged particles



Classical picture:

$$V(x) = \frac{q_p^2\, g^2}{2} x \ ?$$

Coulomb law in 1+1d

||

confinement

too naive in the presence of dynamical fermions

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}\left(1 - e^{-q\mu x}\right)$$

screening

- massive case:

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$$\mu \equiv g/\sqrt{\pi}$$

$$V(x) = \frac{q_p^2 \, g^2}{2\mu}\left(1 - e^{-q\mu x}\right) \qquad \textit{screening}$$

- massive case: [cf. Misumi-Tanizaki-Unsal '19]

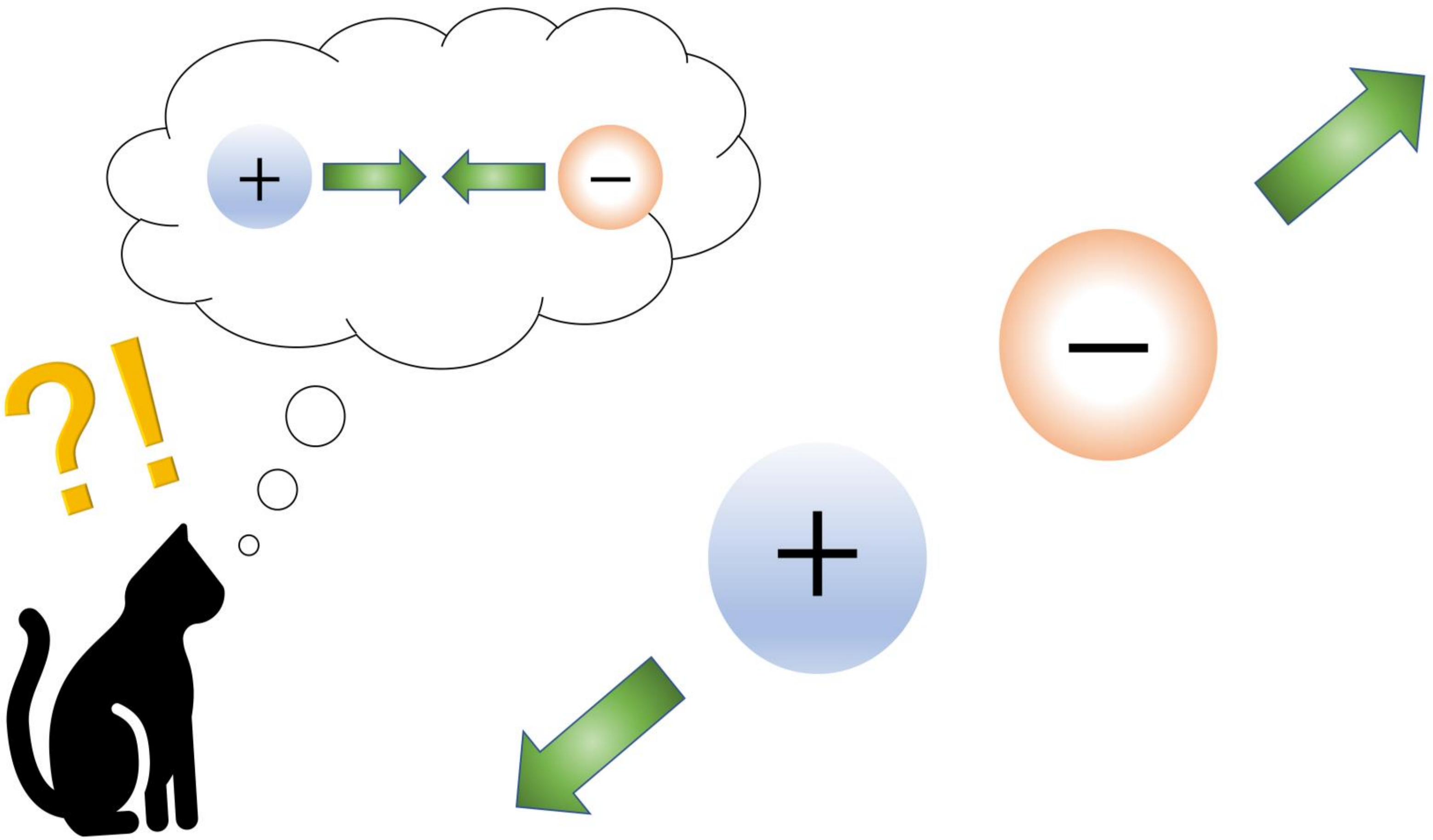
$$\Sigma \equiv g e^{\gamma}/2\pi^{3/2}$$

$$V(x) \sim mq\Sigma\left(\cos\left(\frac{\theta + 2\pi q_p}{q}\right) - \cos\left(\frac{\theta}{q}\right)\right) x \qquad (\mathrm{m} \ll g,\ |x| \gg 1/g\ )$$

$$\begin{cases} = \text{Const.} & \text{for } \mathrm{q_p/q} = \mathbf{Z} \quad \textit{screening} \\ \propto x & \text{for } \mathrm{q_p/q} \neq \mathbf{Z} \quad \textit{confinement?} \end{cases}$$

*but sometimes **negative** slope!*

That is, as changing the parameters…

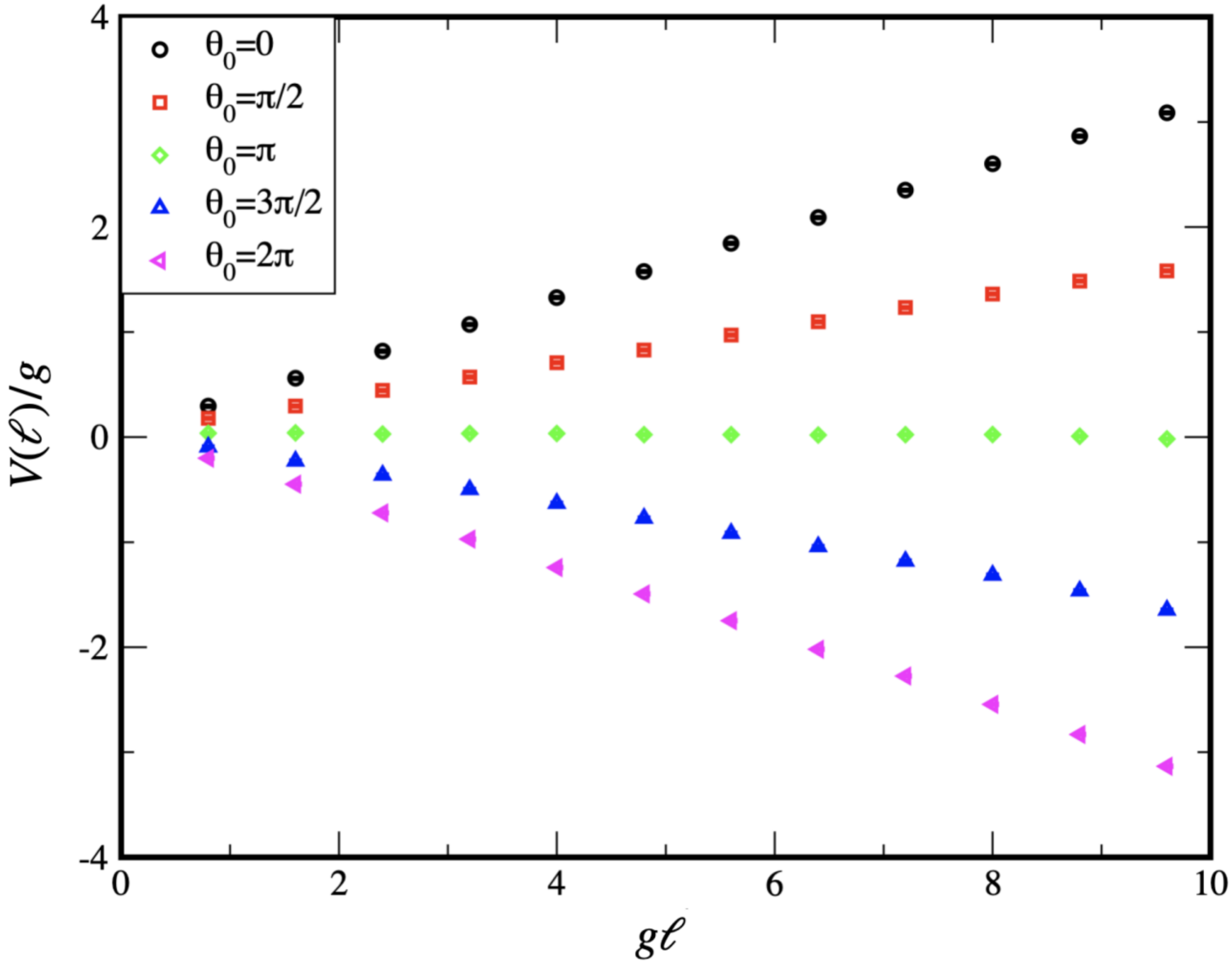


Let's explore this aspect by quantum simulation!

Positive / negative string tension

[MH-Itou-Kikuchi-Tanizaki '21]

Parameters: $g = 1, a = 0.4, N = 25, T = 99, q_p/q = -1/3, m = 0.15$

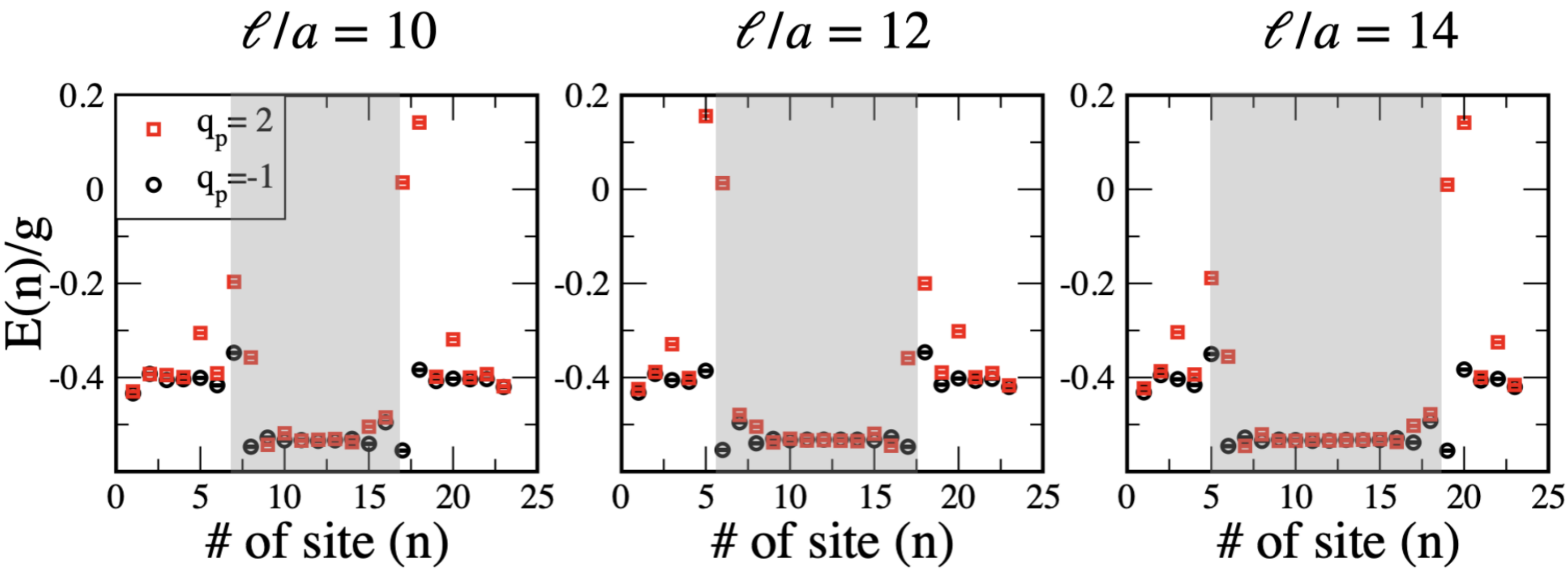


Sign(tension) changes as changing θ-angle!!

Energy density @ negative tension regime

[MH-Itou-Kikuchi-Tanizaki '21]

$$g = 1, a = 0.4, N = 25, T = 99,\ q_p/q = -1/3, m = 0.15, \theta_0 = 2\pi$$



Lower energy inside the probes!!

Towards "quantum supremacy"?

The problems in this talk involve only ground state in 1+1D

$\rightarrow$ Tensor Network is better $\rightarrow$ able to take $N = \mathcal{O}(100)$

[MH-Itou-Tanizaki '22]

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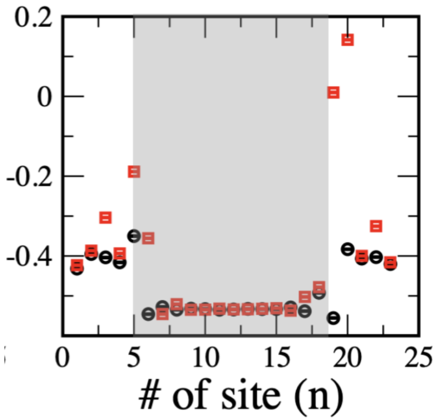
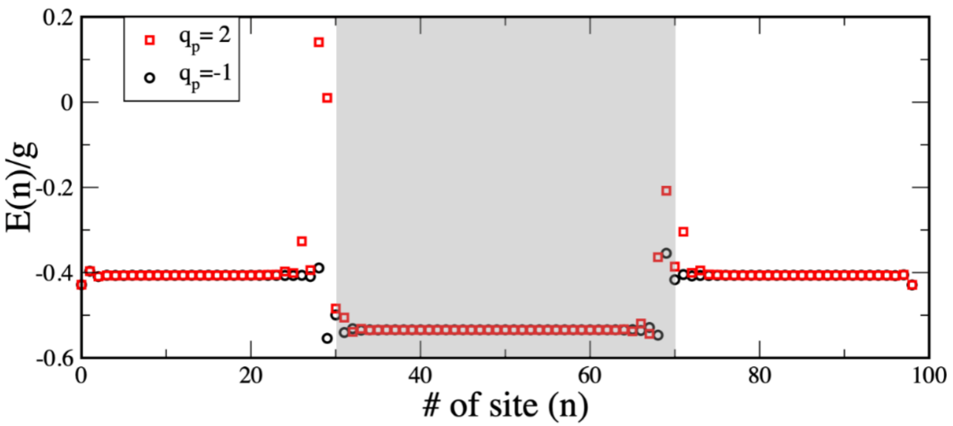
Adiabatic state preparation:

$\ell/a = 14$



Tensor Network (DMRG):

$\ell/a = 40, \quad N = 101$



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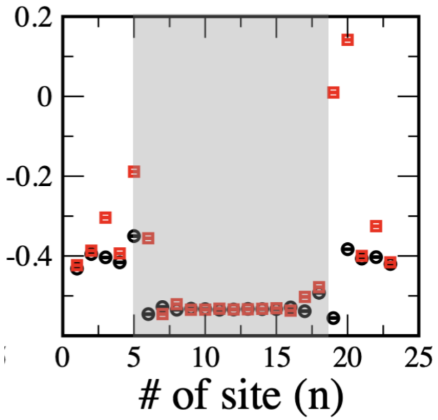

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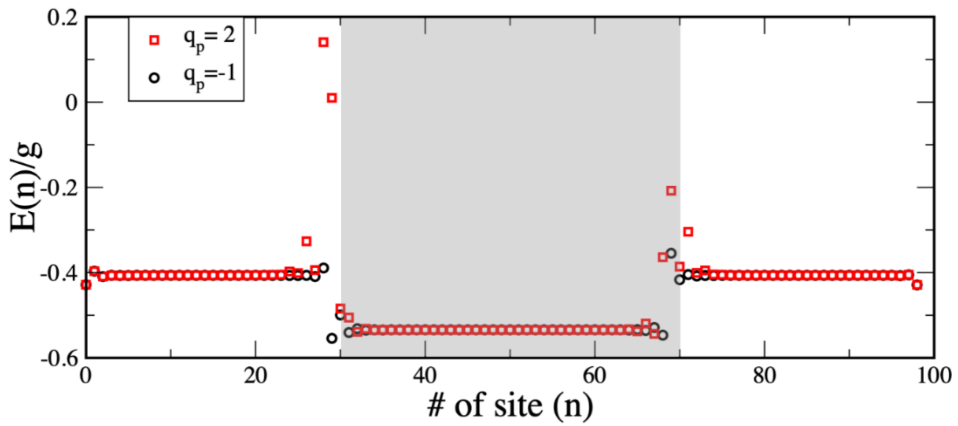

should study problems not efficiently simulated by MC & TN

- long time evolution, many pt. function, non-local op.
- system w/ strong entanglement (matrix models?)

Other simulations of Schwinger model

- decay of massive vacuum under time evolution

  [cf. Martinez etal. **Nature** 534 (2016) 516-519]

- quenched dynamics of θ [Nagano-Bapat-Bauer ’23]

- Schwinger model in open quantum system

  [De Jong-Metcalf-Mulligan-Ploskon-Ringer-Yao ’20, de Jong-Lee-Mulligan-Ploskon-Ringer-Yao ’21, Lee-Mulligan-Ringer-Yao ’23]

- 100 qubit simulation of Schwinger model

  [Farrell-Illa-Ciavarella-Savage ’23]

- finding energy spectrum [MH-Ghim, work in progress]

- finite temperature [Itou-Sun-Pedersen-Yunoki, work in progress]

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Energy spectrum in quantum field theory

Information in energy spectrum:

- degeneracy of ground states
- energy gap between ground & 1st excited states
- distribution of excited states at low levels

⇨ phase structure, mass spectrum of particles

Energy spectrum in quantum field theory

Information in energy spectrum:

- degeneracy of ground states
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⇨ phase structure, mass spectrum of particles

Desired algorithm:

efficient computation of spectrum at low levels

(doesn’t need ground state energy itself)

For this purpose, it seems inefficient to explicitly construct energy eigenstates one by one and measure their energies

Algorithm: coherent imaging spectroscopy

[Senko-Smith-Richerme-Lee-Campbell-Monroe '14]

[working in progress, MH-Ghim]

We'd like to know spectrum of excited energies:

$$\hat{H}_{\text{target}}\,|n\rangle = E_n|n\rangle$$

Time dependent Hamiltonian:

$$\hat{H}(t;\nu) = \hat{H}_{\text{target}} + B\sin(\nu t)\cdot\hat{O}$$

Survival probability of ground state after some time:

$$P(\nu) \coloneqq |\langle 0|\mathcal{T}e^{-i\int dt\hat{H}\,(t;\nu)}|0\rangle|^2$$

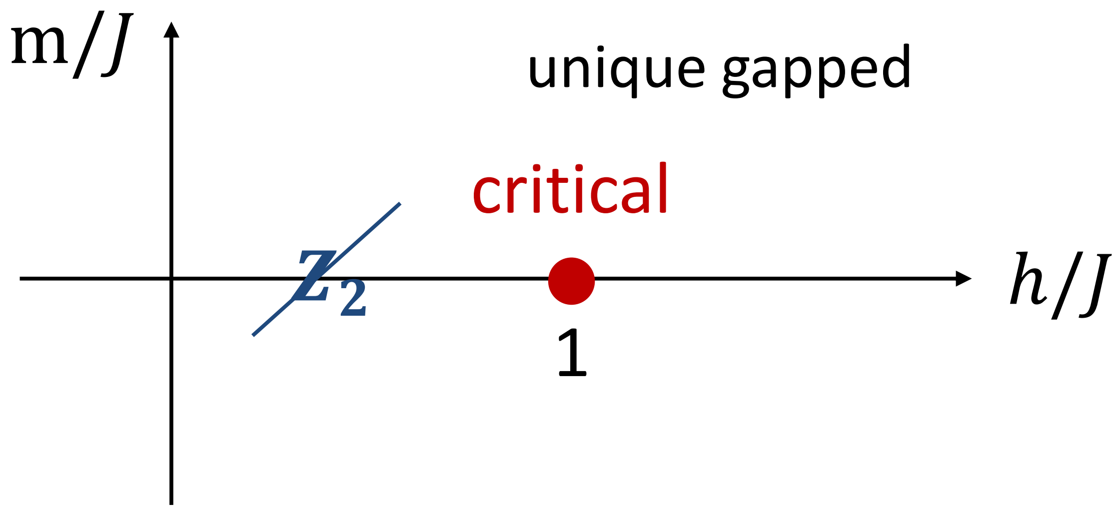
becomes small when $\nu \sim E_n$

Coherent imaging spectroscopy in Ising model

[working in progress, MH-Ghim]

$$\hat{H}_{\text{Ising}} = -J\sum_{n=1}^{N-1} Z_n Z_{n+1} - h\sum_{n=1}^{N} X_n - m\sum_{n=1}^{N} Z_n$$

Known phase diagram:



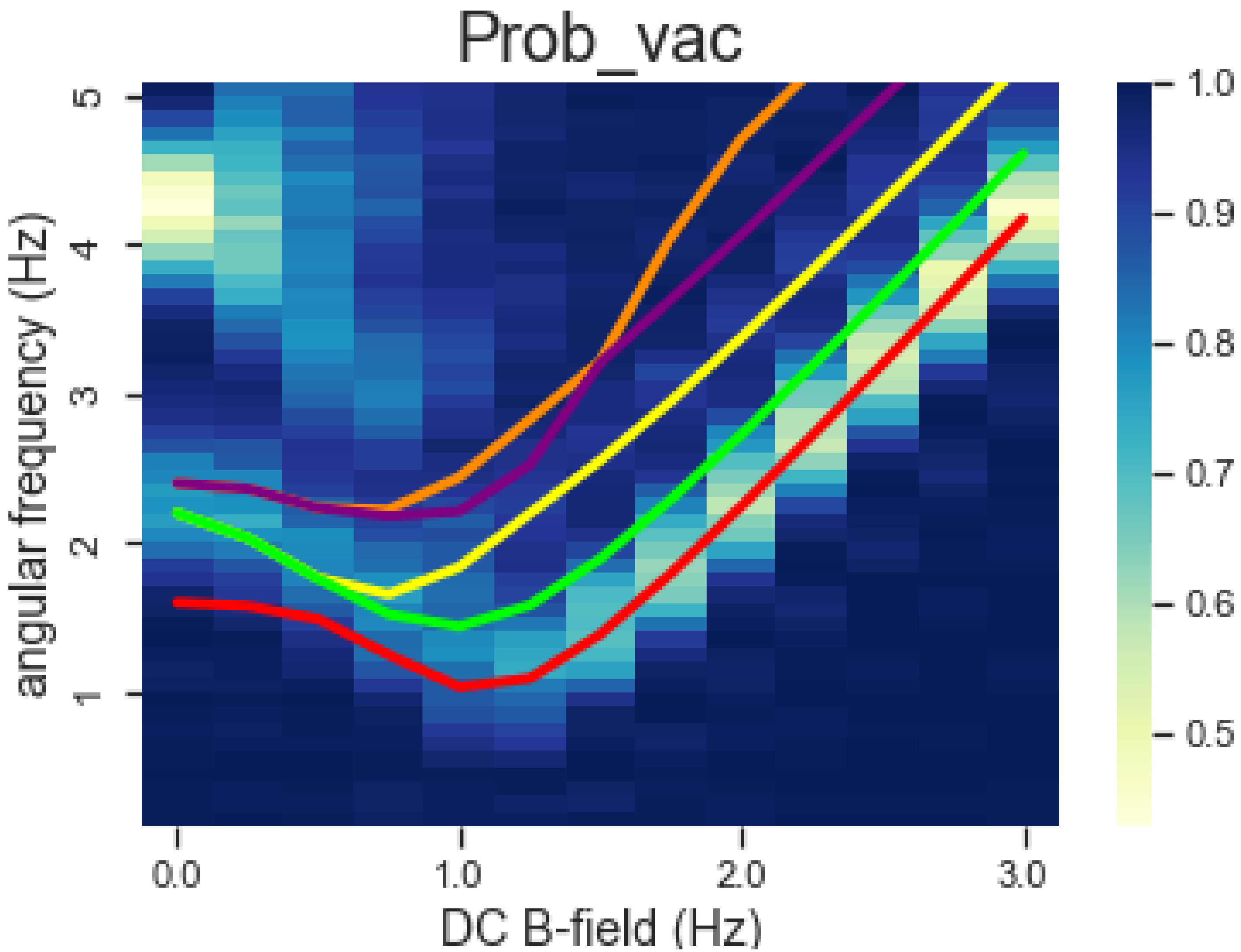
Let's consider time evolution by

$$\hat{H}_{\text{Ising}} + B\sin(\nu t)\sum_{n=1}^{N} Y_n$$

Coherent imaging spectroscopy in Ising model (cont'd)

[working in progress, MH-Ghim]

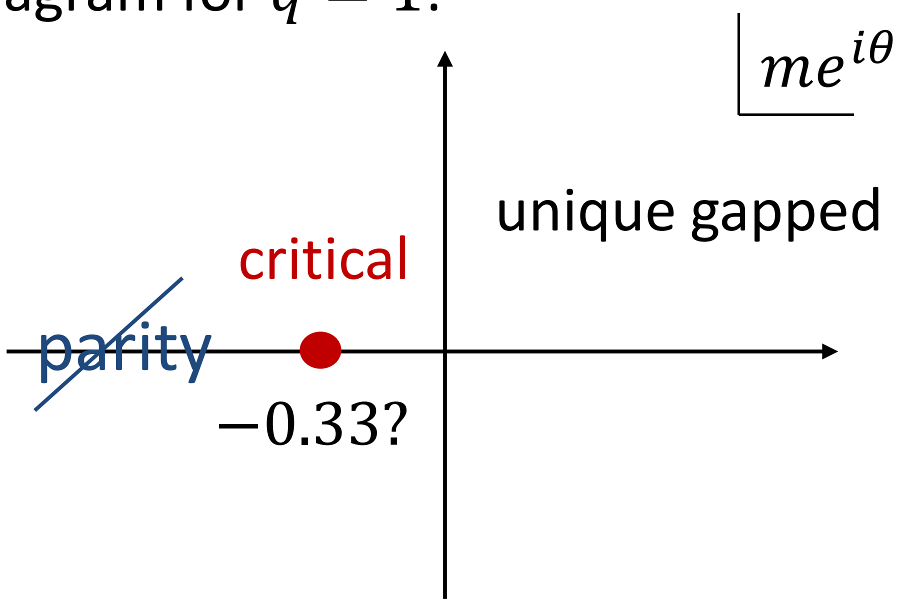
$N = 8, m/J = 0.1$ ($|0\rangle$ by adiabatic state preparation)



Coherent imaging spectroscopy in Schwinger model

$$H = J\sum_{n=0}^{N-2}\left(L_n + \frac{\theta_0}{2\pi}\right)^2 - \mathrm{i}w\sum_{n=0}^{N-2}\left[\chi_n^\dagger (U_n)^q \chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=0}^{N-1}(-1)^n \chi_n^\dagger \chi_n$$

Expected phase diagram for $q = 1$:

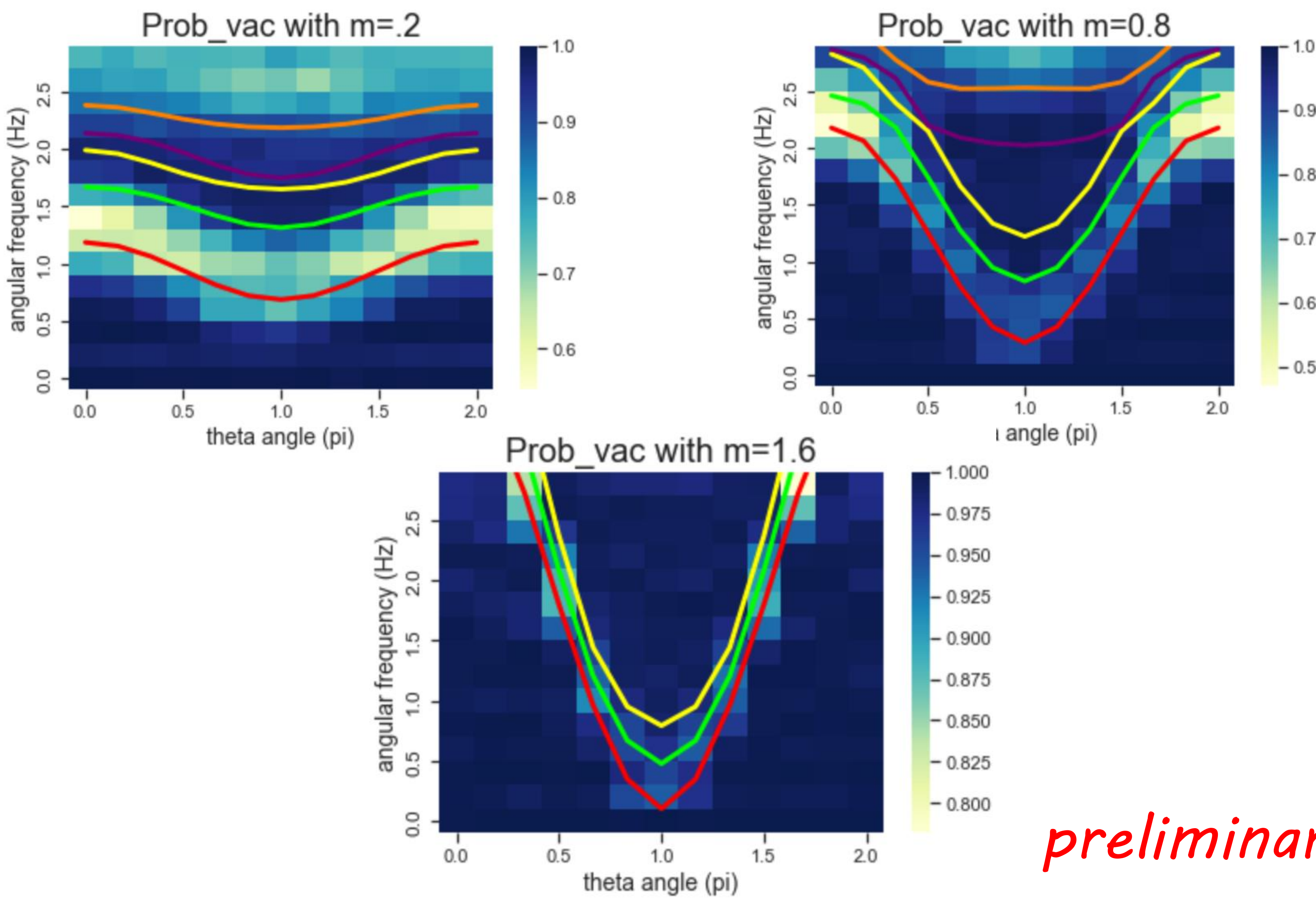


Let's consider time evolution by (perturbed by "$\bar{\psi}\gamma_5\psi$")

$$\hat{H} + B\sin(\nu t)\sum_{n=0}^{N-1}(-1)^n\left(\chi_n^\dagger \chi_{n+1} - \chi_{n+1}^\dagger \chi_n\right)$$

Coherent imaging spectroscopy in Schwinger model (cont'd)

($N = 13, g = 1, w = 1, |0\rangle$ by adiabatic state preparation)



preliminary

On higher dimensional fermion

[MH, work in progress]

Go to higher dimensions!

$\Downarrow$

1st step: find a nice way to map 2d fermion to spins

Problem in naïve approach:

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i\right)$$

- 1d

$$\chi_{n+1}^{\dagger}\chi_n \xrightarrow{\text{Jordan-Wigner}} \exists\, X_{n+1}X_n, Y_{n+1}Y_n, X_{n+1}Y_n, Y_{n+1}X_n$$

local

- 2d

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local

- 2d ($N \times N$ square lattice)

Relabeling site (i, j) like 1d label (say $n = i + Nj$),

$$\chi_{(i,j+1)}^{\dagger}\chi_{(i,j)} = \chi_{I+N}^{\dagger}\chi_I \xrightarrow{\text{JW}} {}^{\exists}X_{I+N}X_I \prod_{i=I+1}^{I+N-1} Z_i \text{ , etc...}$$

(cf. $\mathcal{O}(\log N)$ for Bravyi-Kitaev trans.)

non-local

Application of a new map to field theory

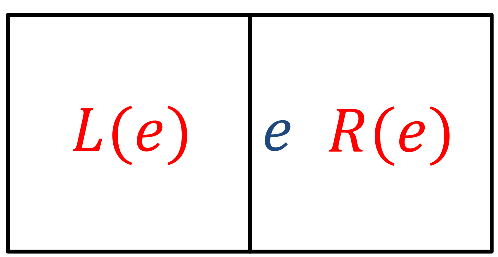
[Chen-Kapustin-Radicevic '17]

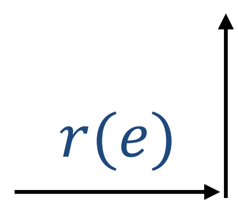
2 Majorana fermions on face $\Longleftrightarrow$ Spin op. on edge

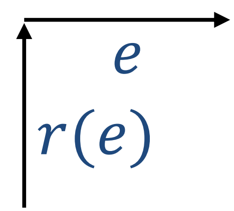
$$(-1)^{F_f} = -i\gamma_f\gamma'_f \longleftrightarrow W_f. \qquad S_e = i\gamma_{L(e)}\gamma'_{R(e)} \longleftrightarrow U_e$$

where $$W_f = \prod_{e\subset f} Z_e. \qquad U_e = X_e Z_{r(e)}.$$

"Gauss law" constraint at site v: $$W_{\mathrm{NE}(v)} \prod_{e\supset v} X_e = 1.$$







ex.)

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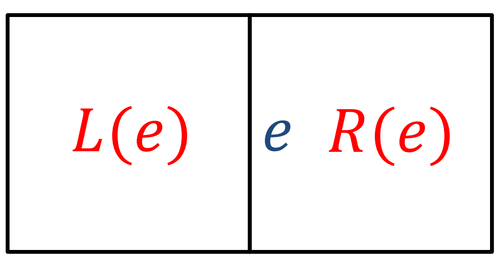

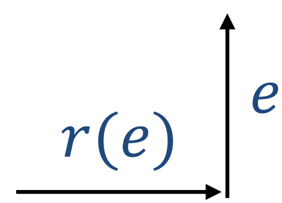

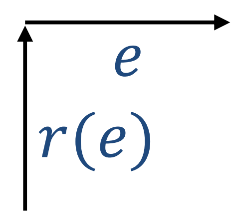

ex.) $$H = t\sum_e (c^\dagger_{L(e)}c_{R(e)} + c^\dagger_{R(e)}c_{L(e)}) + \mu\sum_f c^\dagger_f c_f.$$

$$\Longrightarrow \quad H = \frac{t}{2}\sum_e X_e Z_{r(e)}(1 - W_{L(e)}W_{R(e)}) + \frac{\mu}{2}\sum_f (1 - W_f)$$

local

Some other applications

- Scattering [Jordan-Lee-Preskiill ’17]
- Inflation (scalar in curved spacetime) [Liu-Li ’20]
- Efficient simulation of (2+1)d U(1) gauge th. [Kane-Grabowska-Nachman-Bauer ’22]
- Chiral fermion [Hayata-Nakayama-Yamamoto ’23]
- Quantum group approach to Non-abelian gauge th. [Zache-Gonzalez-Cuadra-Zoller ’23, Hayata-Hidaka ’23]
- Dark sector showers [Chigusa-Yamazaki ’22, Bauer-Chigusa-Yamazaki ’23]
- Measurement-based quantum computation [Okuda-Sukeno ’22]
- quantum machine learning [Nagano-Miessen-Onodera-Tavernelli-Tacchino-Terashi ’23]
- String/M-theory [Gharibyan-Hanada-MH-Liu ’20]

etc...

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Quantum simulation is a promising approach if $\exists$ much computational resource in future

Challenges:

- to get sufficient # of qubits to implement quantum error correction (QEC)
- to identify efficient ways to put gauge theory on quantum computers

This talk:

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This talk:

relations between QEC & gauge theory

relations between QEC & gauge theory

Motivations

[Spirit may be similar to Rajput-Roggaro-Wiebe ’21, Gustafson-Lamm ’23, etc...]

(some points elaborated later)

1. $^{\exists}$explicit examples

   ex.) Toric code = $\boldsymbol{Z_2}$ lattice gauge theory [Kitaev ’97]

2.

3.

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relations between QEC & gauge theory

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$$\begin{cases} \text{QEC} = \text{redundant description of logical qubits} \\ \text{Gauge theory} = \text{redundant description of physical states} \end{cases}$$

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3. Nature = Gauge theory & Nature = Quantum computer

$\Longrightarrow$ Gauge theory may know something on QEC?

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4. $^\exists$proposals on relations among QEC & concepts in HEP

   ex.) Holography, Black hole, CFT, Renormalization group

   [Almheiri-Dong-Harlow ’14, Hayden-Preskill ’07, Dymarsky-Shapere ’20, Kawabata-Nishioka-Okuda ’22, Furuya-Lashkari-Moosa ’21, etc...]

What I'm doing...

[MH, work in progress]

to make dictionary for classes of codes/gauge theories:

QEC	Gauge theory
errors	unphysical op. (& excitation)
logical qubits	physical states (w/ low energy)
"no error conditions" (stabilizer)	Gauss law (& min[energy])
logical op.	gauge invariant op.
ancilla for recovery	additional matter
⋮	⋮

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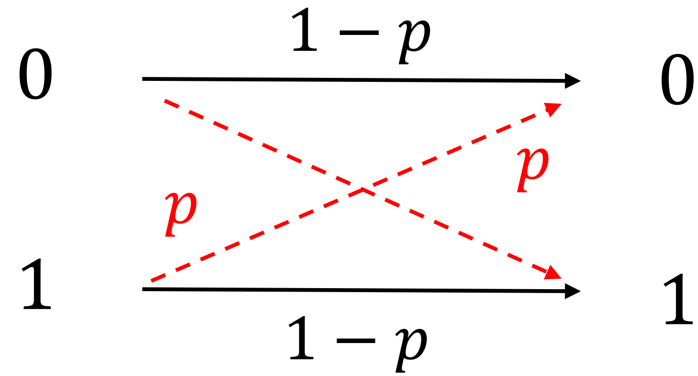
Errors in classical computers

Computer interacts w/ environment ⟹ error/noise

one bit



Suppose we send a bit but have “error” in probability p

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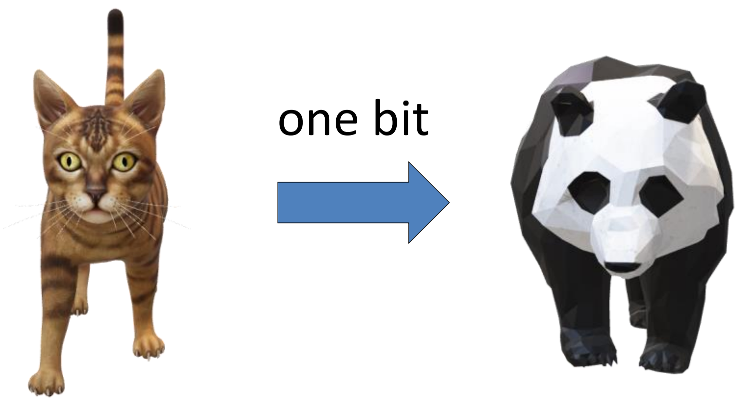

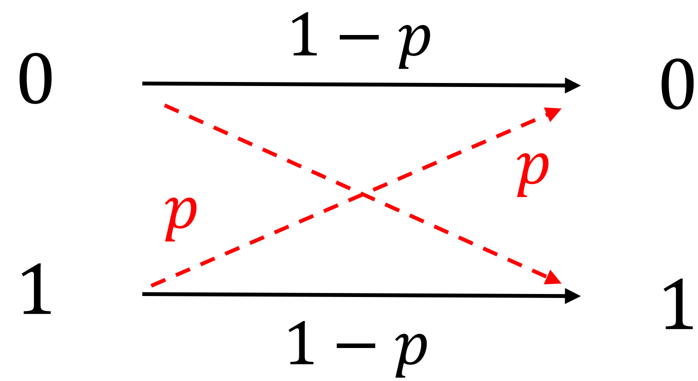

Suppose we send a bit but have "error" in probability p

A simple way to correct errors:

① Duplicate the bit (encoding): $0 \rightarrow 000, \quad 1 \rightarrow 111$

② Error detection & correction by "majority voting":

$$001 \rightarrow 000, \quad 011 \rightarrow 111, \quad \text{etc...}$$

➡ $P_{\text{failed}} = 3p^2(1-p) + p^3$ (improved if $p < 1/2$)

Errors in quantum computers

Computer interacts w/ environment $\Rightarrow$ error/noise

- Unknown unitary operators are multiplied:

(in addition to decoherence & measurement errors)

$$|\psi\rangle \xrightarrow{\text{error!}} U|\psi\rangle$$

not only bit flip!

Errors in quantum computers

Computer interacts w/ environment $\Rightarrow$ error/noise

- Unknown unitary operators are multiplied:

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$$|\psi\rangle \xrightarrow{\text{error!}} U|\psi\rangle$$

not only bit flip!

- have to detect errors & act "inverse of errors" to recover w/o destroying states

- need more qubits as in the classical case

Ex.) 3-qubit bit flip code

Bit flip error

$$|\psi\rangle \to \quad X|\psi\rangle \qquad \text{w/ probability } p$$

Encoding

Error detection

Ex.) 3-qubit bit flip code

Bit flip error

$$|\psi\rangle \rightarrow \quad X|\psi\rangle \qquad \text{w/ probability } p$$

Encoding

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle \quad \longrightarrow \quad |\psi_E\rangle = c_0|000\rangle + c_1|111\rangle$$

Error detection

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Error detection

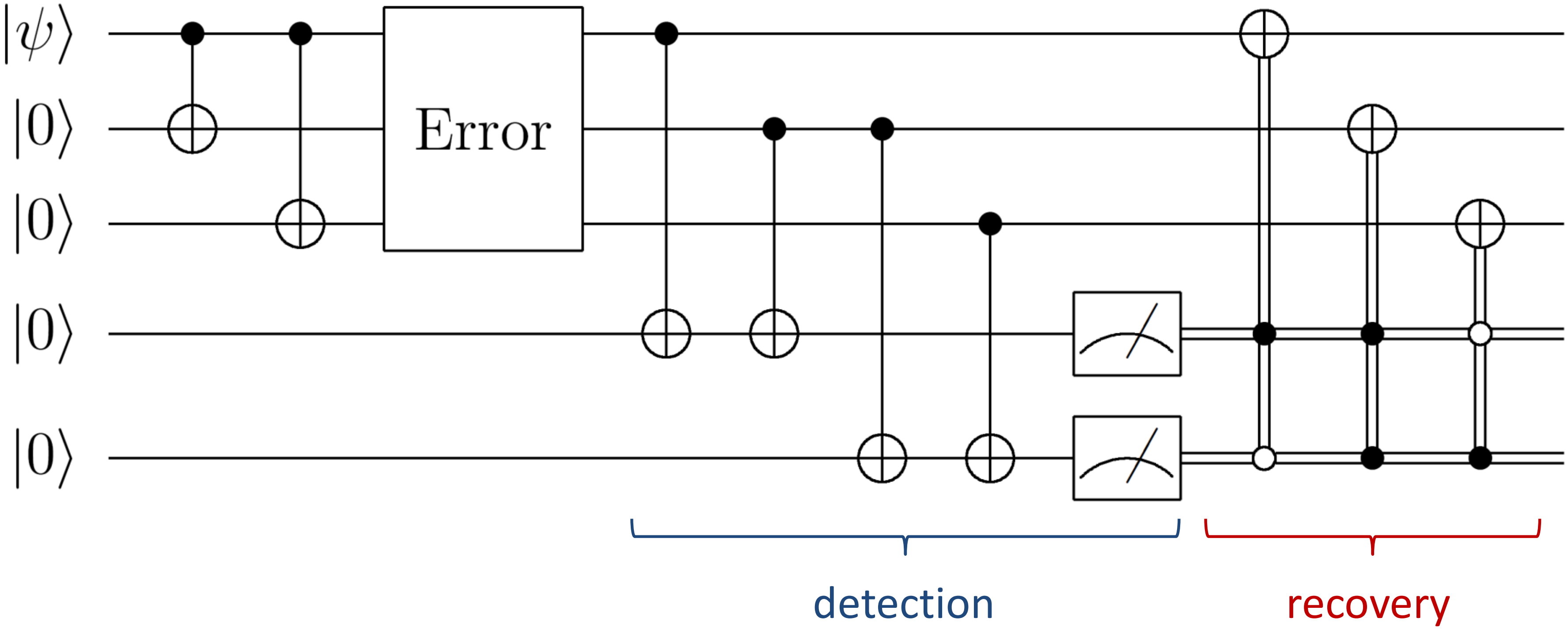
If error occurs once, we can detect the error by knowing

$$Z_1Z_2 \;\; \& \;\; Z_2Z_3$$

"No error" condition:

$$(Z_1Z_2)|\psi_E\rangle = |\psi_E\rangle\,, \qquad (Z_2Z_3)|\psi_E\rangle = |\psi_E\rangle$$

Error recovery in 3-qubit bit flip code



As in the classical case, it fails if $^{\exists}$multiple "errors":

$$P_{\text{failed}} = 3p^2(1-p) + p^3 \qquad \text{(improved if } p < 1/2\text{)}$$

Quantum Error Correction

1.Encoding

$$|\psi\rangle \in \mathcal{H} \longrightarrow |\psi_E\rangle \in \mathcal{H}_E \quad (\mathcal{H} \subset \mathcal{H}_E)$$

2. Error detection

Take set of operators $\{O_1, \cdots\}$ s.t.

$$O_i|\psi_E\rangle = |\psi_E\rangle, \quad O_i(\mathrm{error})|\psi_E\rangle \neq (\mathrm{error})|\psi_E\rangle$$

Then find eigenvalues of O_i's using ancillary qubits

3. Error recovery

Act “inverse of error” based on the eigenvalues

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Conceptual similarity?

Quantum error correction:

description of logical qubits by more qubits

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Gauge theory:

Conceptual similarity?

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Ex.) 3-qubit bit flip code

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Gauge theory:

description of physical states by larger state space

Ex.) $U(1)$ gauge theory + matters

$$\nabla \cdot \hat{\boldsymbol{E}}(x)|\mathrm{phys}\rangle = \hat{\rho}(x)|\mathrm{phys}\rangle$$ *"Gauss law"*

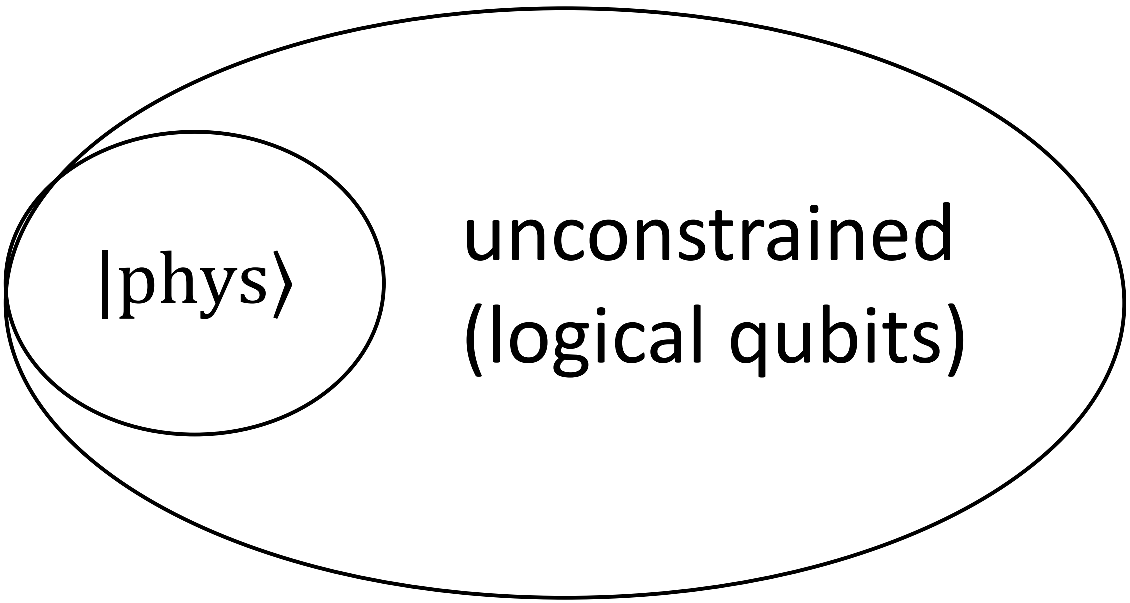
Gauge theory on QC w/ error correction

When we don't solve Gauss law before simulation…

$|\mathrm{phys}\rangle$

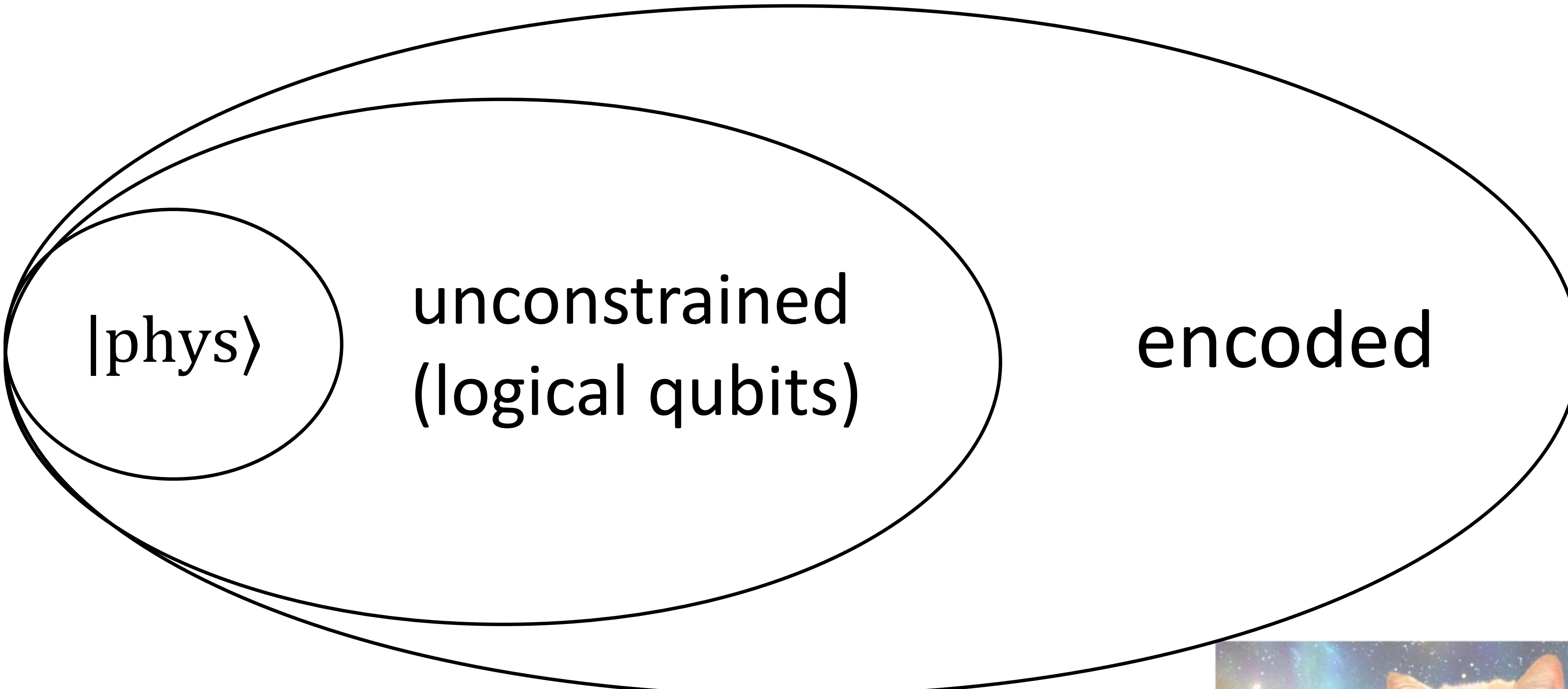
Gauge theory on QC w/ error correction

When we don't solve Gauss law before simulation…



Gauge theory on QC w/ error correction

When we don't solve Gauss law before simulation…



redundancy^2 !!

Gauge theory on QC w/ error correction (cont'd)

Could we avoid the redundancy^2 ??

Possible hints:

$$\begin{cases} \text{Nature} = \text{quantum computer} \\ \text{Nature} = \text{gauge theory} \end{cases}$$

⇨ quantum computer = gauge theory ??

Gauge theory knows something on error correction?

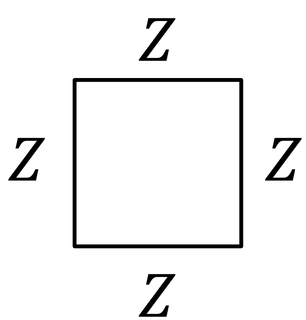
(I don't have a clear answer at this moment
but I'm trying to make connections precise)

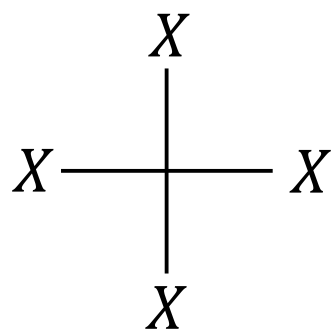
Ex.) Toric code

[Kitaev '97]

Consider 2d periodic square lattice and put qubits on edges

$$H = -J \sum_{\text{face}} \prod_{e \in \partial(\text{face})} Z_e - J \sum_{\text{vertex}} \prod_{e | \partial e = \text{vertex}} X_e$$



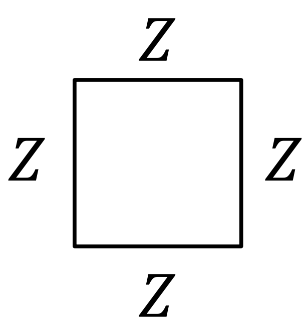


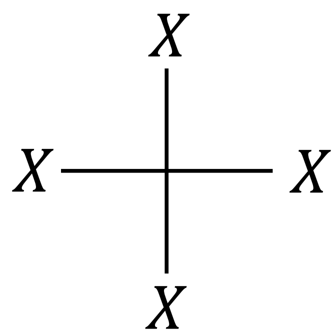
Ex.) Toric code

[Kitaev ’97]

Consider 2d periodic square lattice and put qubits on edges

$$H = -J \sum_{\text{face}} \prod_{e \in \partial(\text{face})} Z_e - J \sum_{\text{vertex}} \prod_{e | \partial e = \text{vertex}} X_e$$





“No error” condition = minimum energy condition:

$$\prod\nolimits_{e \in \partial(\text{face})} Z_e \, |\psi_E\rangle = |\psi_E\rangle, \quad \prod\nolimits_{e | \partial e = \text{vertex}} X_e \, |\psi_E\rangle = |\psi_E\rangle$$

$\Rightarrow$ logical op. = products of X, Z along nontrivial cycles

Ex.) Toric code (cont'd)

$\boldsymbol{Z_2}$ gauge theory on 2d square lattice: $\quad (U \sim e^{iA}, \Pi \sim e^{iE} \in \boldsymbol{Z_2})$

$$H = g^2 \sum_e \Pi_e - J \sum_{\text{face}} \prod_{e \in \partial(\text{face})} U_e$$

$$(\Pi_e U_{e'} \Pi_e^\dagger = -\delta_{ee'} U_e)$$

Ex.) Toric code (cont'd)

$\boldsymbol{Z_2}$ gauge theory on 2d square lattice: $\quad (U \sim e^{iA}, \Pi \sim e^{iE} \in \boldsymbol{Z_2})$

$$H = g^2 \sum_e \Pi_e - J \sum_{\text{face}} \prod_{e \in \partial(\text{face})} U_e$$

<u>Gauss law:</u> $\quad (\Pi_e U_{e'} \Pi_e^\dagger = -\delta_{ee'} U_e)$

$$\prod_{e | \partial e = \text{vertex}} \Pi_e \, |\text{phys}\rangle = |\text{phys}\rangle$$

Ex.) Toric code (cont'd)

$\boldsymbol{Z_2}$ gauge theory on 2d square lattice: $(U \sim e^{iA}, \Pi \sim e^{iE} \in \boldsymbol{Z_2})$

$$H = g^2 \sum_e \Pi_e - J \sum_{\text{face}} \prod_{e \in \partial(\text{face})} U_e$$

<u>Gauss law:</u> $(\Pi_e U_{e'} \Pi_e^\dagger = -\delta_{ee'} U_e)$

$$\prod_{e|\partial e = \text{vertex}} \Pi_e \,|\text{phys}\rangle = |\text{phys}\rangle$$

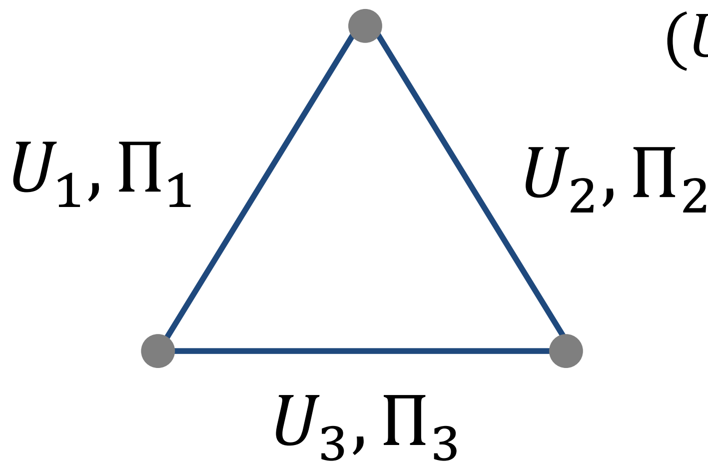
<u>Ground state for $g = 0$:</u>

$$\prod\nolimits_{e|\partial e = \text{vertex}} U_e \,|\text{ground}\rangle = |\text{ground}\rangle$$

In identification (U-basis)~(computational basis),
this is the same condition as the toric code

Ex.) $\boldsymbol{Z_2}$ lattice gauge theory on 3 sites

$(U \sim e^{iA}, \Pi \sim e^{iE} \in \boldsymbol{Z_2})$



Hamiltonian:

$(\Pi_m U_n \Pi_m^\dagger = -\delta_{mn} U_n)$

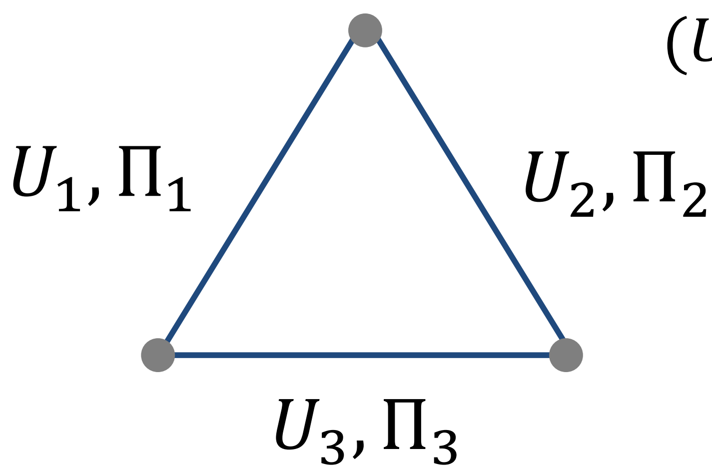
$$H = -J \sum_{n=1}^{3} \left(\Pi_n + \Pi_n^\dagger \right)$$

Gauss law:

$$\Pi_n \Pi_{n-1}^\dagger |\mathrm{phys}\rangle = |\mathrm{phys}\rangle$$

Ex.) $\boldsymbol{Z_2}$ lattice gauge theory on 3 sites (cont'd)

$(U \sim e^{iA}, \Pi \sim e^{iE} \in \boldsymbol{Z_2})$



$$\boxed{\Pi_n \Pi_{n-1}^{\dagger} |\mathrm{phys}\rangle = |\mathrm{phys}\rangle}$$

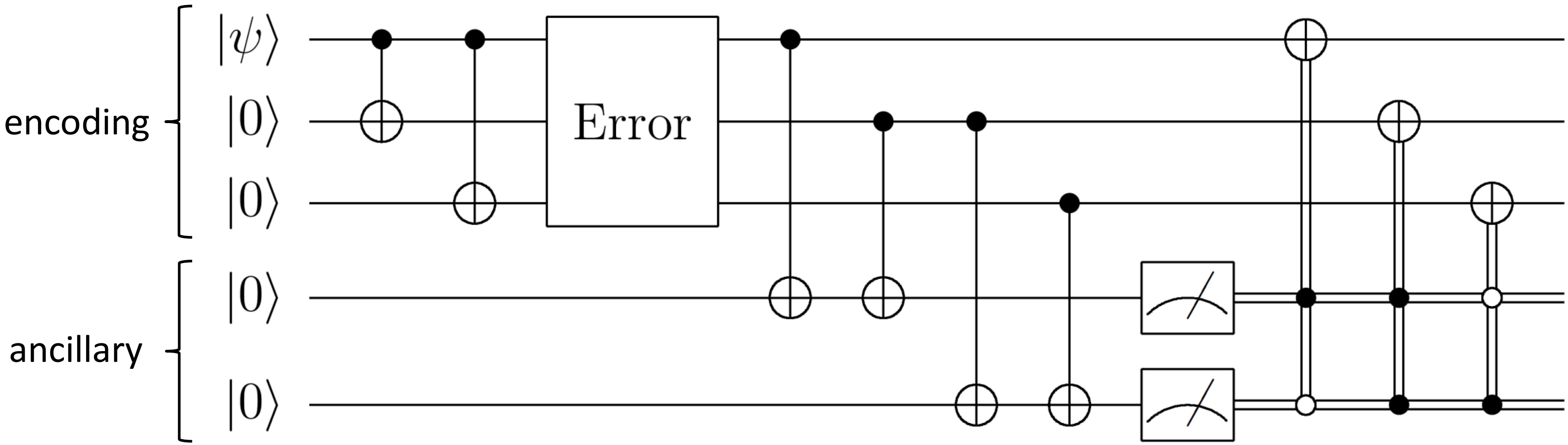
Taking (computational basis) $\sim$ (eigenstate of Π_n)

⇩

$$Z_1 Z_2 |\mathrm{phys}\rangle = |\mathrm{phys}\rangle\,, \quad Z_2 Z_3 |\mathrm{phys}\rangle = |\mathrm{phys}\rangle$$

"no error" condition in 3-qubit bit flip code!

Error detection & recovery

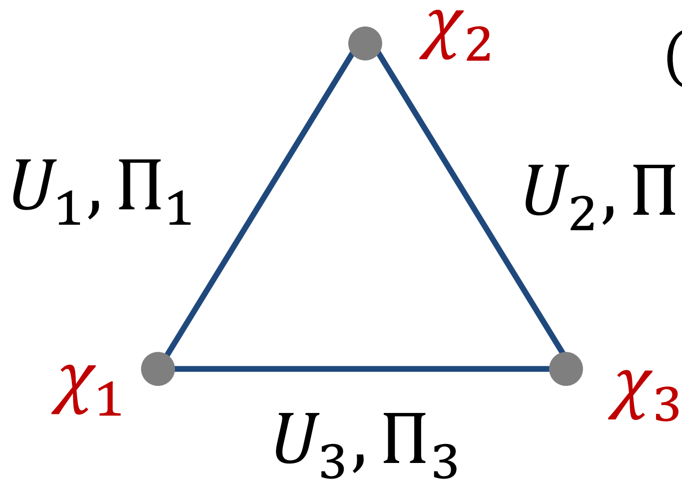


Is there analogue of this in gauge theory?

— Ancilla may be matter on sites (next slide)

$\boldsymbol{Z_2}$ lattice gauge theory w/ a complex fermion

$(U \sim e^{iA}, \Pi \sim e^{iE} \in \boldsymbol{Z_2})$



Hamiltonian:

$$H = -J \sum_{n=1}^{3} \left(\Pi_n + \Pi_n^\dagger\right) + w \sum_{n=1}^{3} (\chi_{n+1}^\dagger U_n \chi_n - \chi_n^\dagger U_n^\dagger \chi_{n+1})$$

Commutation relation:

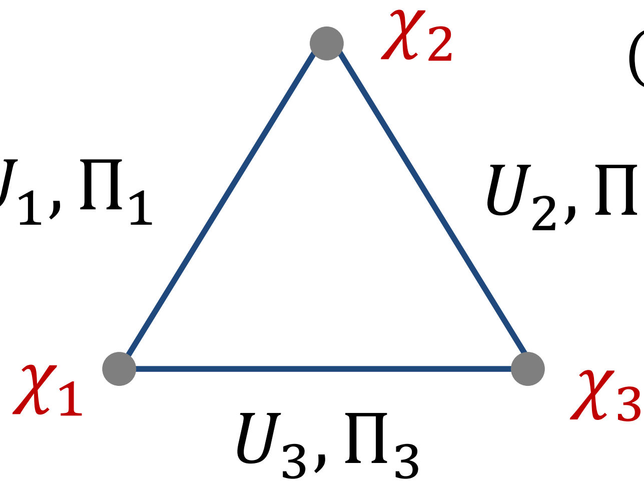
$$\Pi_m U_n \Pi_m^\dagger = -\delta_{mn} U_n \qquad \left\{\chi_m, \chi_n^\dagger\right\} = \delta_{mn}$$

Gauss law:

$$\Pi_n \Pi_{n-1}^\dagger |\text{phys}\rangle = e^{i\pi\chi_n^\dagger \chi_n} |\text{phys}\rangle$$

$\boldsymbol{Z_2}$ lattice gauge theory w/ a complex fermion (cont'd)

$(U \sim e^{iA}, \Pi \sim e^{iE} \in \boldsymbol{Z_2})$



$$\boxed{\Pi_n \Pi_{n-1}^\dagger |\mathrm{phys}\rangle = e^{i\pi \chi_n^\dagger \chi_n} |\mathrm{phys}\rangle}$$

Taking (computational basis) $\sim$ (eigenstate of Π_n)

$$\Downarrow$$

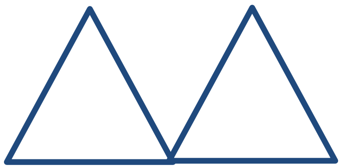
$$Z_1 Z_2 |\mathrm{phys}\rangle = e^{i\pi \chi_n^\dagger \chi_n} |\mathrm{phys}\rangle\,, \quad Z_2 Z_3 |\mathrm{phys}\rangle = e^{i\pi \chi_n^\dagger \chi_n} |\mathrm{phys}\rangle$$

Measuring Fermion charge = Syndrome measurement?

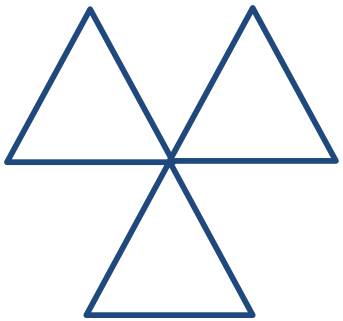
Some generalizations

[MH, work in progress]

- $\boldsymbol{Z_2}$ theory on 1d periodic lattice w/ $(2n+1)$ sites = $[2n+1, 1, 2n+1]$ code



$= [6,2,3],$



$= [9,3,3], \cdots$

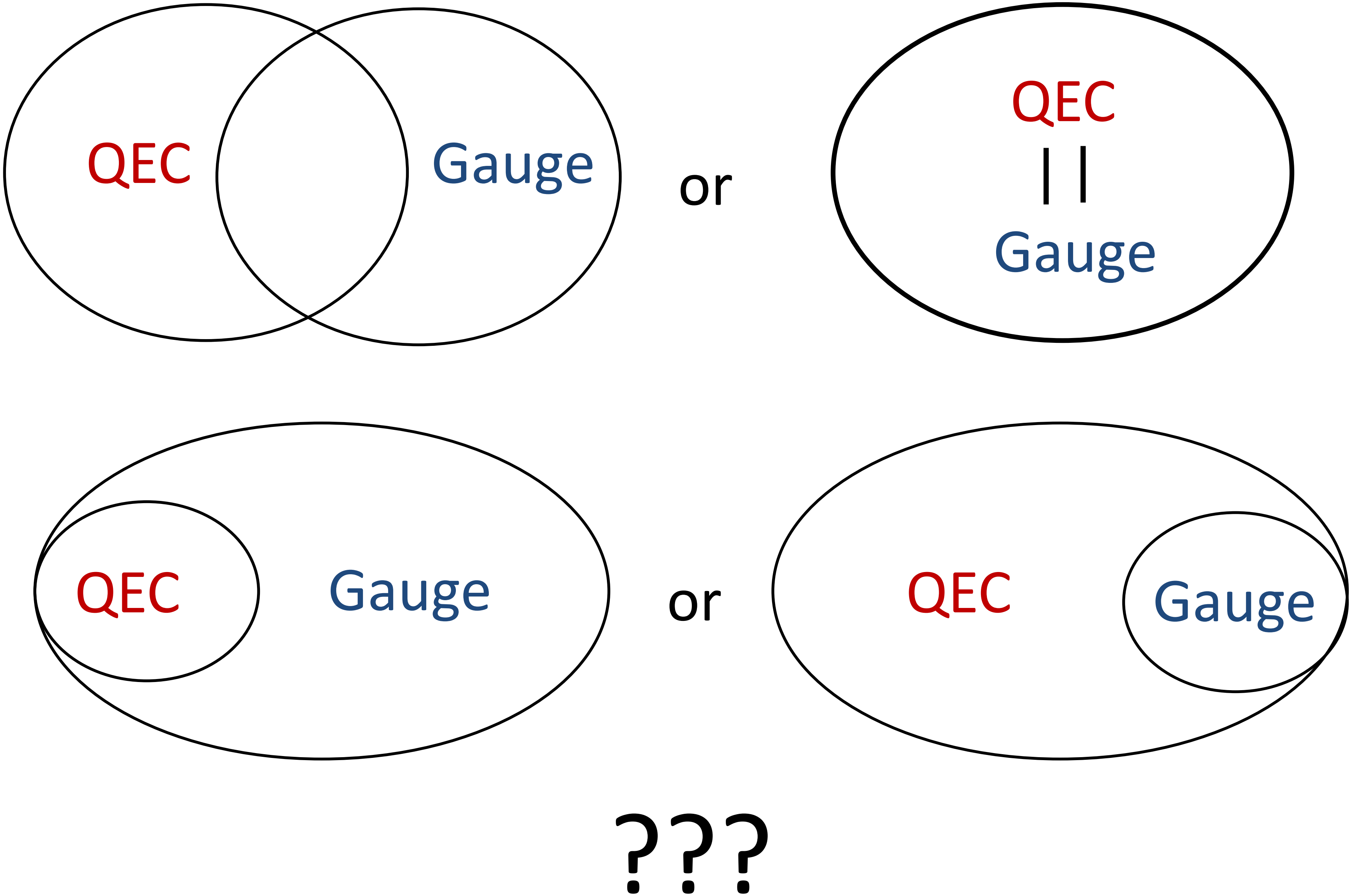
- Phase flip code is done by changing basis
- Shor code seems to need products of plaquettes
- $\boldsymbol{Z_2} \to \boldsymbol{Z_N}$ makes qubit qudit w/ $d = N$
- 5-qubit perfect code is a special case of variant of toric code [Bonilla Ataides etal. '20]

Summary

[MH, work in progress]

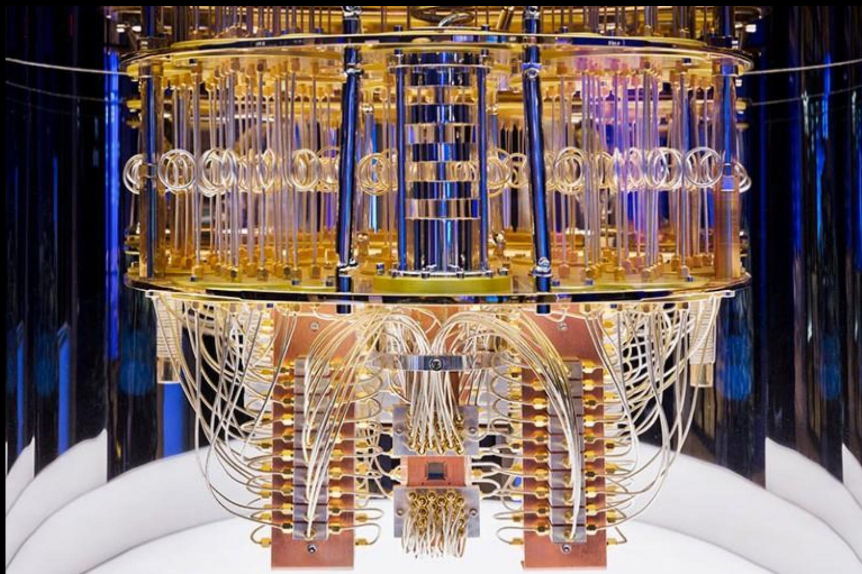
"*QEC/Gauge correspondence*"

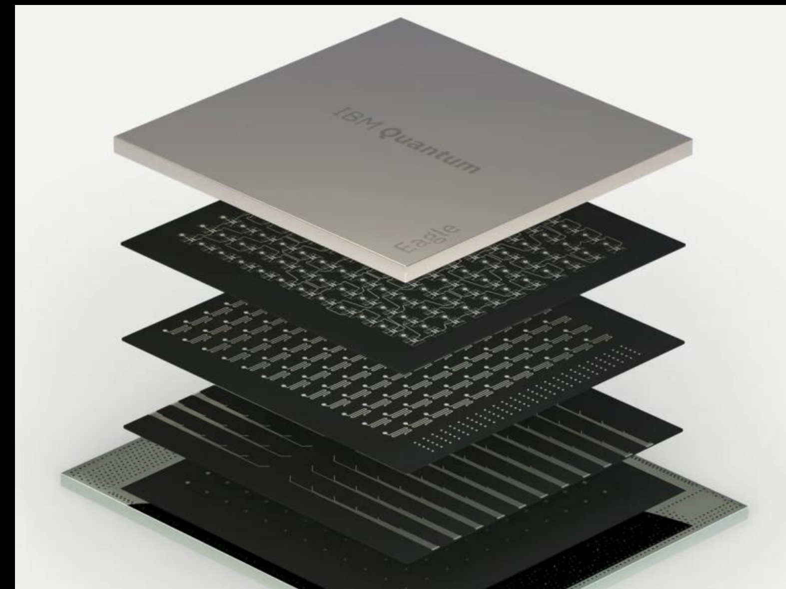
QEC	Gauge theory
errors	unphysical op. (& excitation)
logical qubits	physical states (w/ low energy)
"no error conditions" (stabilizer)	Gauss law (& min[energy])
logical op.	gauge invariant op.
ancilla for recovery	additional matter
⋮	⋮


QEC
Gauge
or
QEC
||
Gauge
QEC
Gauge
or
QEC
Gauge
???

Outlook

The challenge by IBM’s 127-qubit device





Article

Evidence for the utility of quantum computing before fault tolerance

https://doi.org/10.1038/s41586-023-06096-3

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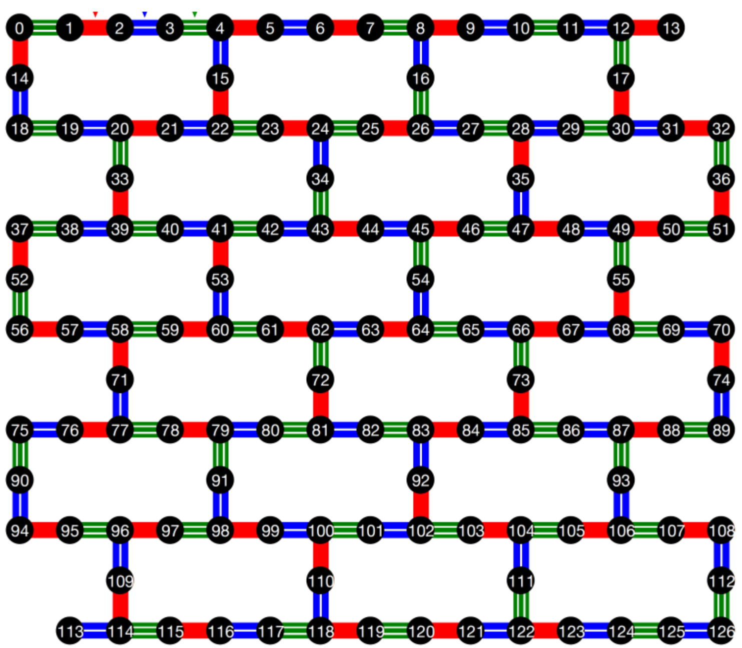
Youngseok Kim[1,6]✉, Andrew Eddins[2,6]✉, Sajant Anand[3], Ken Xuan Wei[1], Ewout van den Berg[1], Sami Rosenblatt[1], Hasan Nayfeh[1], Yantao Wu[3,4], Michael Zaletel[3,5], Kristan Temme[1] & Abhinav Kandala[1]✉

Quantum computing promises to offer substantial speed-ups over its classical

The challenge by IBM's 127-qubit device (cont'd)

Task: time evolution of Ising model on a lattice
w/ shape = the qubit config. of the device



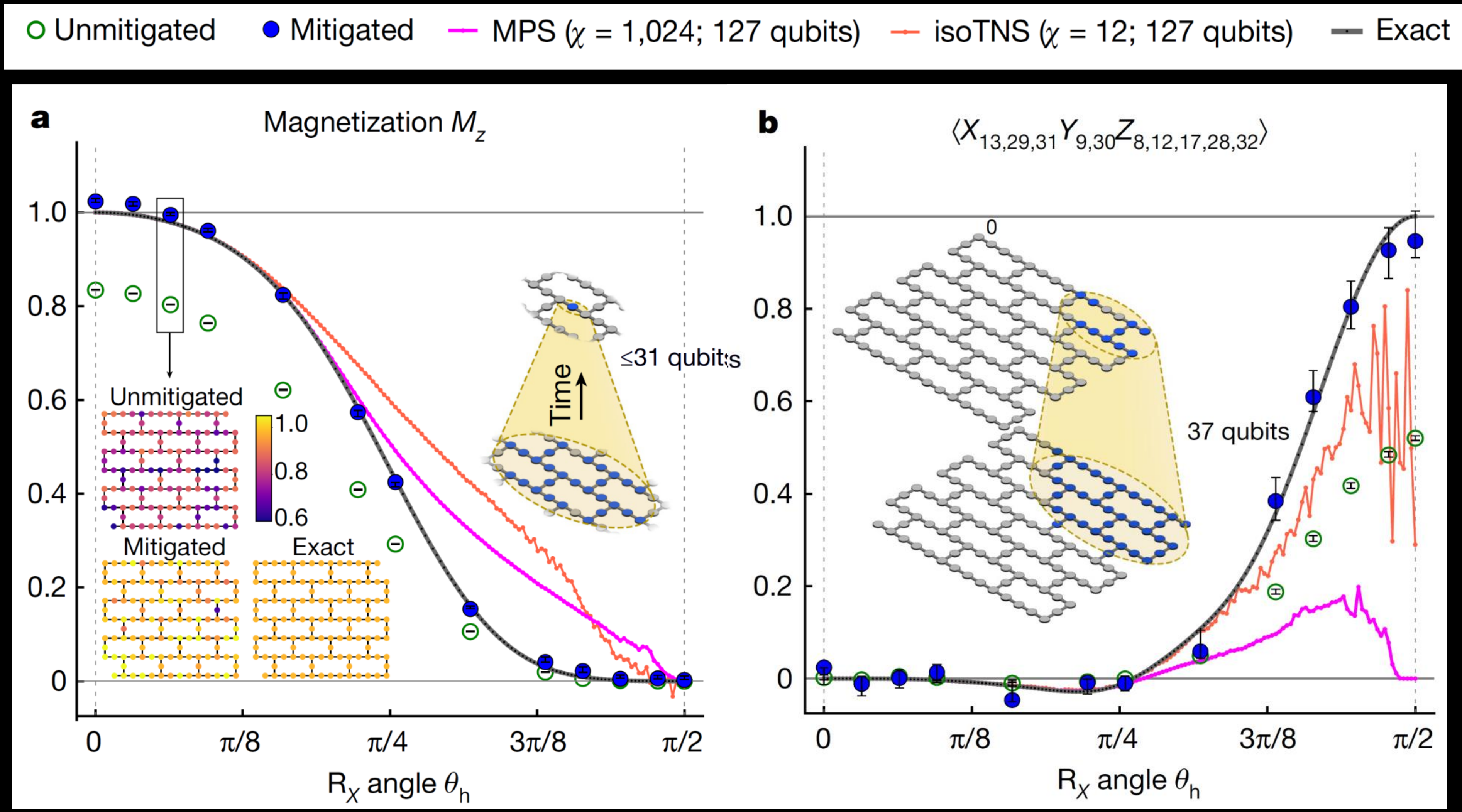
$$H = -J \sum_{\langle i,j \rangle} Z_i Z_j + h \sum_i X_i,$$

$$|\psi(t)\rangle := e^{-iHt} |00\cdots 0\rangle$$

$$\langle \psi(t) | \mathcal{O} | \psi(t) \rangle$$

Strategy: Suzuki-Trotter approximation
+ error mitigation by extrapolation

The challenge by IBM's 127-qubit device (cont'd)



"Quantum supremacy"?

But…

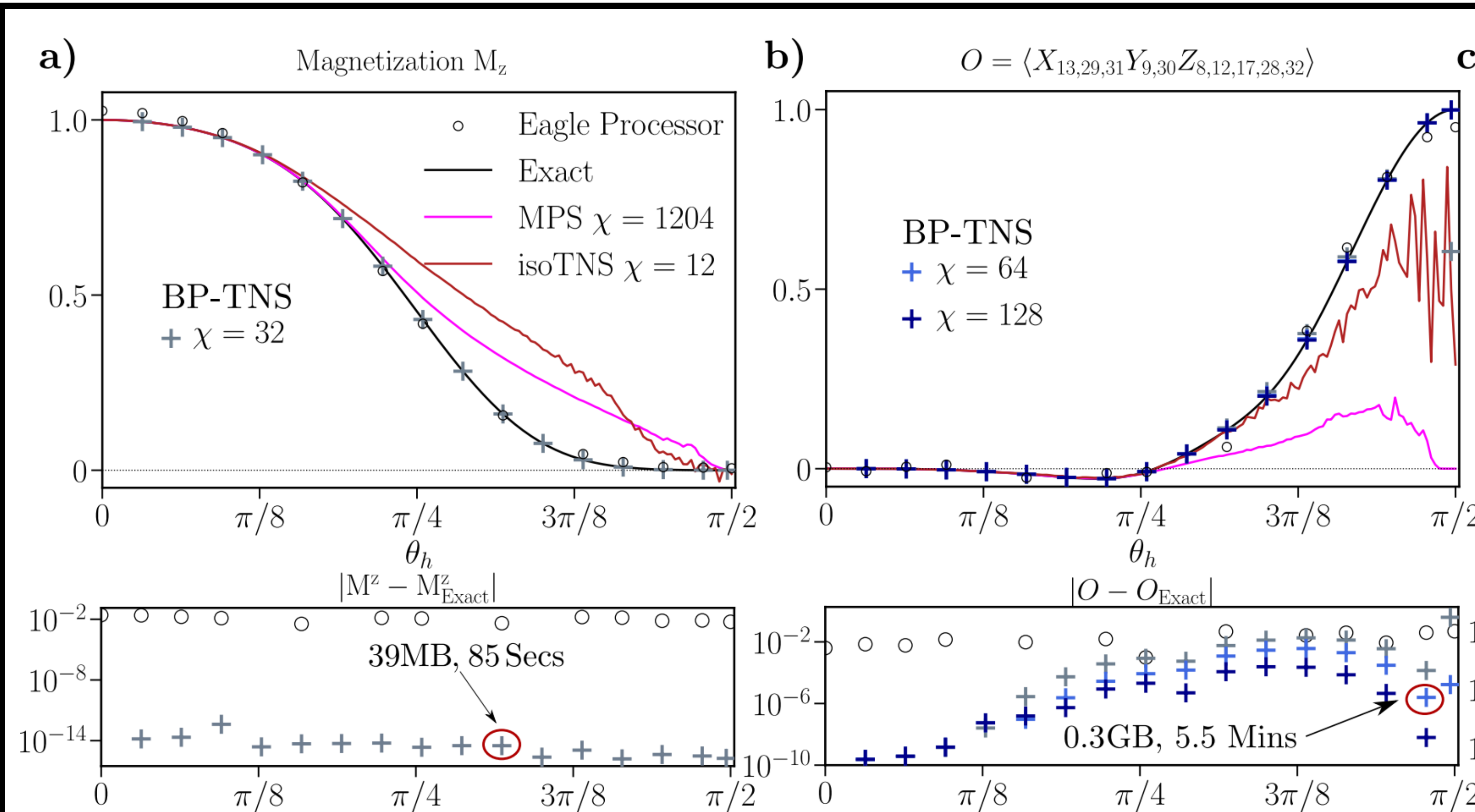
arXiv > quant-ph > arXiv:2306.14887

Quantum Physics

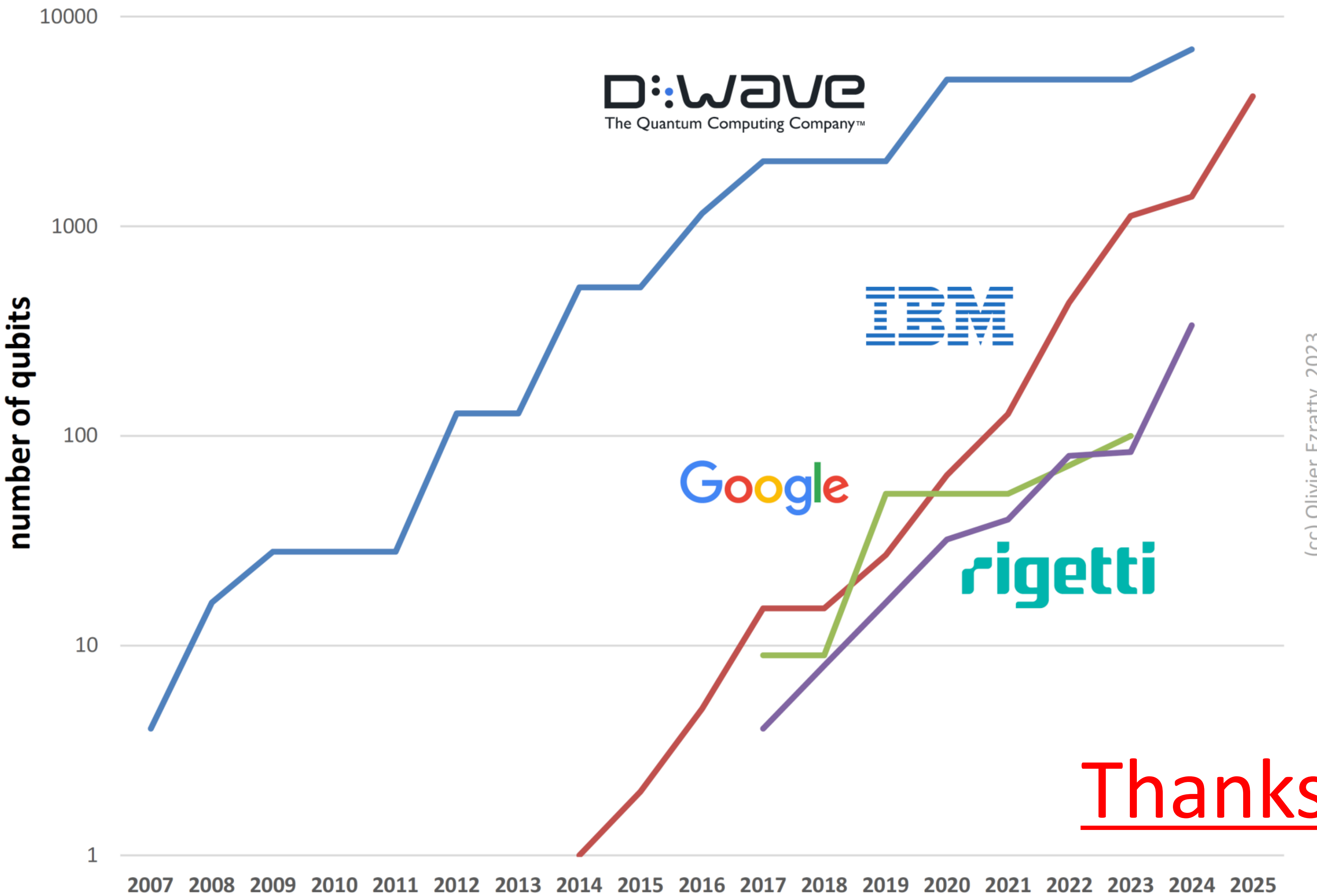
[Submitted on 26 Jun 2023]

Efficient tensor network simulation of IBM's kicked Ising experiment

Joseph Tindall, Matt Fishman, Miles Stoudenmire, Dries Sels



"Quantum" Moore's law?



Thanks!

Appendix

Sign problem in Monte Carlo simulation

Conventional approach to simulate QFT:

① Discretize Euclidean spacetime by lattice:

& make path integral finite dimensional:

$$\int D\phi \ \mathcal{O}(\phi) e^{-S[\phi]} \quad \Rightarrow \quad \int d\phi \ \mathcal{O}(\phi) e^{-S(\phi)}$$

② Numerically Evaluate it by (Markov Chain) Monte Carlo method regarding the Boltzmann factor as a probability:

$$\langle \mathcal{O}(\phi) \rangle \simeq \frac{1}{\sharp(\text{samples})} \sum_{i \in \text{samples}} \mathcal{O}(\phi_i)$$

Sign problem in Monte Carlo simulation (Cont'd)

Markov Chain Monte Carlo:

$$\int d\phi\ \mathcal{O}(\phi)\underbrace{e^{-S(\phi)}}_{\text{probability}}$$

problematic when Boltzmann factor isn't $\mathbf{R}_{\geqq 0}$ & is highly oscillating

Examples w/ sign problem:

- topological term —— complex action
- chemical potential —— indefinite sign of fermion determinant
- real time —— " $e^{iS(\phi)}$ " *much worse*

In operator formalism,

sign problem is absent from the beginning

($^{\exists}$various approaches within framework of path integral formalism but I'll skip it)

Schwinger model

Accessible region by analytic computation

- Massive limit:

The fermion can be integrated out

&

the theory becomes effectively pure Maxwell theory w/ θ

- Bosonization:

[Coleman '76]

$$\mathcal{L} = \frac{1}{8\pi}\left(\partial_\mu \phi\right)^2 - \frac{g^2}{8\pi^2}\phi^2 + \frac{e^\gamma g}{2\pi^{3/2}} m \cos(\phi + \theta)$$

exactly solvable for $m = 0$

&

small m regime is approximated by perturbation

Symmetries in charge-q Schwinger model

$$L = \frac{1}{2g^2}F_{01}^2 + \frac{\theta_0}{2\pi}F_{01} + \overline{\psi}\,\mathrm{i}\,\gamma^\mu(\partial_\mu + \mathrm{i}\,q\,A_\mu)\psi - m\,\overline{\psi}\psi$$

- $\boldsymbol{Z_q}$ chiral symmetry for $m = 0$

—— ABJ anomaly: $U(1)_A \to \boldsymbol{Z_q}$

—— known to be spontaneously broken

- $\boldsymbol{Z_q}$ 1-form symmetry

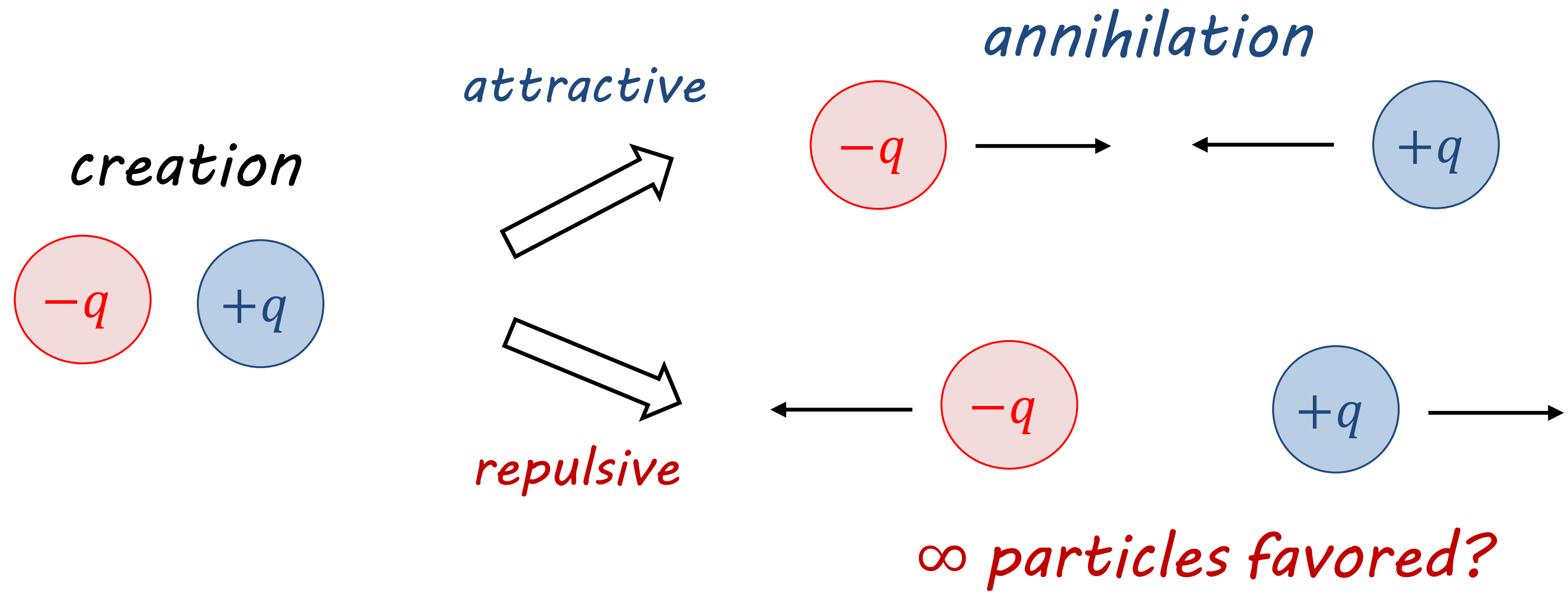
—— remnant of $U(1)$ 1-form sym. in pure Maxwell

—— Hilbert sp. is decomposed into q-sectors *"universe"*

(cf. common for $(d-1)$-form sym. in d dimensions)

FAQs on negative tension behavior

Q1. It sounds that many pair creations are favored.
Is the theory unstable?

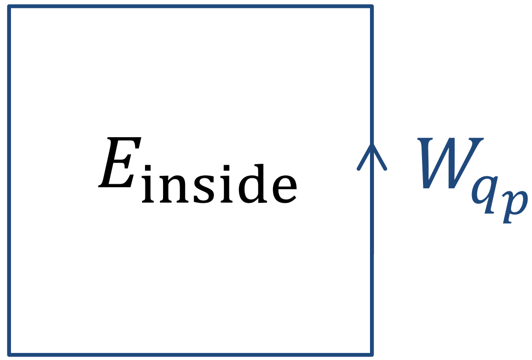


— No. Negative tension appears only for $q_p \neq q\boldsymbol{Z}$.

So, such unstable pair creations do not occur.

FAQs on negative tension behavior (cont'd)

[cf. MH-Itou-Kikuchi-Tanizaki '21]



$E_{\text{outside}}\ (= E_0?\,)$

Q2. It sounds $E_{\text{inside}} < E_{\text{outside}}$. Strange?

— Inside & outside are in different sectors decomposed by Z_q 1-form sym.

$$\mathcal{H} = \bigoplus_{\ell=0}^{q-1} \mathcal{H}_\ell \qquad \text{"universe"}$$

E_{inside} & E_{outside} are lowest in each universe:

$$E_{\text{inside}} = \min_{\mathcal{H}_{\ell+qp}} (E)\,, \qquad E_{\text{outside}} = \min_{\mathcal{H}_\ell} (E)$$

Comment on adiabatic state preparation

$$(\text{"systematic error"}) \sim \frac{1}{T\,(\text{gap})^2}$$

😄 Advantage:

- guaranteed to be correct for $T \gg 1$ & $\delta t \ll 1$ if $H_A(t)$ has a unique gapped vacuum
- can directly get excited states under some conditions

☹️ Disadvantage:

- doesn't work for degenerate vacua
- costly — likely requires many gates

➡ more appropriate for FTQC than NISQ

Without probes

VEV of mass operator (chiral condensation)

$$\langle \bar{\psi}(x)\psi(x)\rangle = \langle \mathrm{vac}|\bar{\psi}(x)\psi(x)|\mathrm{vac}\rangle$$

Instead of the local op., we analyze the average over the space:

$$\frac{1}{2Na}\langle \mathrm{vac}|\sum_{n=1}^{N}(-1)^n Z_n|\mathrm{vac}\rangle$$

Once we get the vacuum, we can compute the VEV as

$$\frac{1}{2Na}\langle \mathrm{vac}|\sum_{n=1}^{N}(-1)^n Z_n|\mathrm{vac}\rangle = \frac{1}{2Na}\sum_{n=1}^{N}(-1)^n \sum_{i_1\cdots i_N=0,1}\langle \mathrm{vac}|Z_n|i_1\cdots i_N\rangle\langle i_1\cdots i_N|\mathrm{vac}\rangle$$

$$= \frac{1}{2Na}\sum_{n=1}^{N}\sum_{i_1\cdots i_N=0,1}(-1)^{n+i_n}\,|\langle i_1\cdots i_N|\mathrm{vac}\rangle|^2$$

How can we obtain the vacuum?

Adiabatic state preparation (cont'd)

$$|\mathrm{vac}> = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\ H_A(t)\right)|\mathrm{vac}_0>$$

$$\simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\mathrm{vac}_0>$$

$$\left(U(t) = e^{-iH_A(t)\delta t}\right)$$

Here, we choose

$$\left\{\begin{array}{l} H_0 = H\Big|_{w\to 0,\, \vartheta_n\to 0,\, m\to m_0} \quad \Rightarrow \quad |\mathrm{vac}_0\rangle = |1010\cdots\rangle \\ H_A(t) = H\Big|_{w\to w(t), \vartheta_n\to\vartheta_n(t),\, m\to m(t)} \\ w(t) = f\left(\frac{t}{T}\right)w,\ \vartheta_n(t) = f\left(\frac{t}{T}\right)\vartheta_n, \quad m(t) = \left(1 - f\left(\frac{t}{T}\right)\right)m_0 + f\left(\frac{t}{T}\right)m \\ f(s)\text{: smooth function s.t. } f(0) = 0,\ f(1) = 1 \end{array}\right.$$

Massless case

For massless case,

θ is absorbed by chiral rotation $\Longrightarrow$ $\theta = 0$ w/o loss of generality

No sign problem

Nevertheless,

it's difficult in conventional approach because computation of fermion determinant becomes very heavy

$^{\exists}$Exact result:

[Hetrick-Hosotani '88]

$$\langle \bar{\psi}(x)\psi(x) \rangle = -\frac{e^{\gamma}}{2\pi^{3/2}} g \simeq -0.160g$$

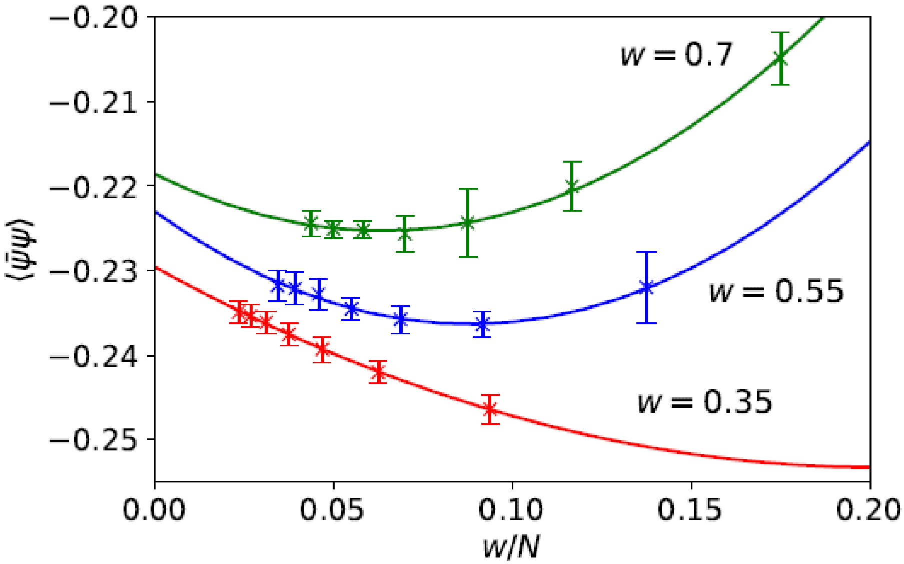
Can we reproduce it?

Thermodynamic & Continuum limit

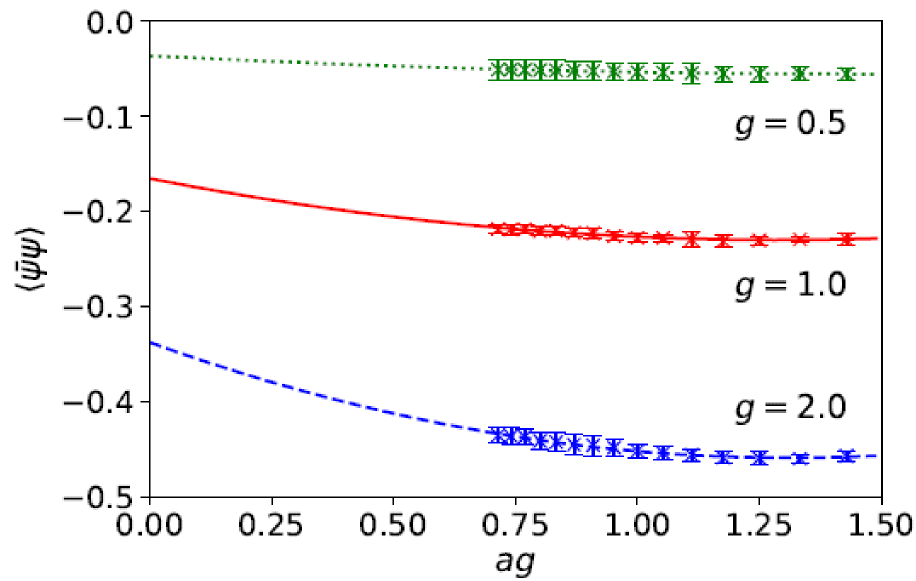
$g = 1, m = 0, N_{\text{max}} = 16, T = 100, \delta t = 0.1, 1M$ shots

#(measurements)

Thermodynamic limit (w/ fixed a)



Continuum limit (after $V \to \infty$)



Estimation of systematic errors

Approximation of vacuum:

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

$$|\text{vac}> \simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\text{vac}_0> \equiv |\text{vac}_A\rangle$$

Approximation of VEV:

$$\langle\mathcal{O}\rangle \equiv \langle\text{vac}|\mathcal{O}|\text{vac}\rangle \simeq \langle\text{vac}_A|\mathcal{O}|\text{vac}_A\rangle$$

Introduce the quantity

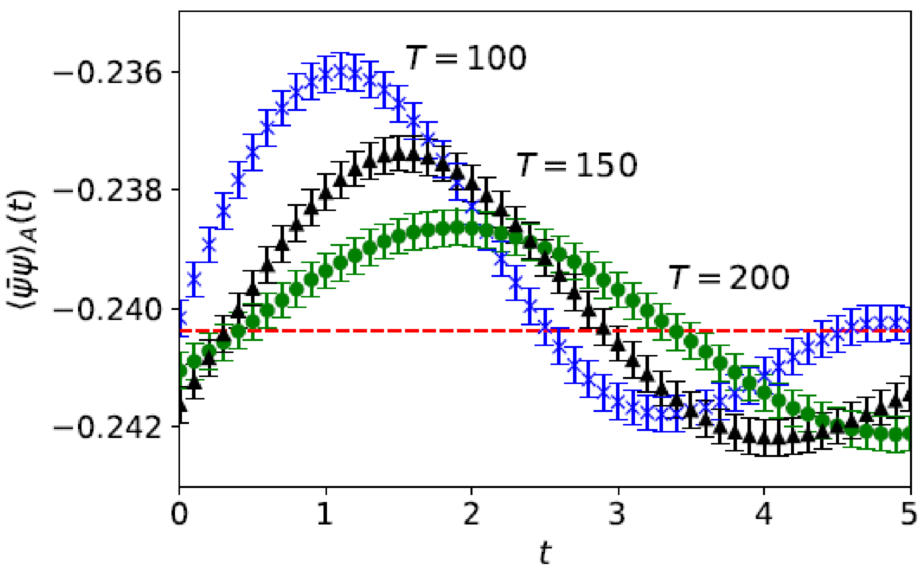
$$\langle\mathcal{O}\rangle_A(t) \equiv \langle\text{vac}_A|e^{i\hat{H}t}\mathcal{O}e^{-i\hat{H}t}|\text{vac}_A\rangle$$

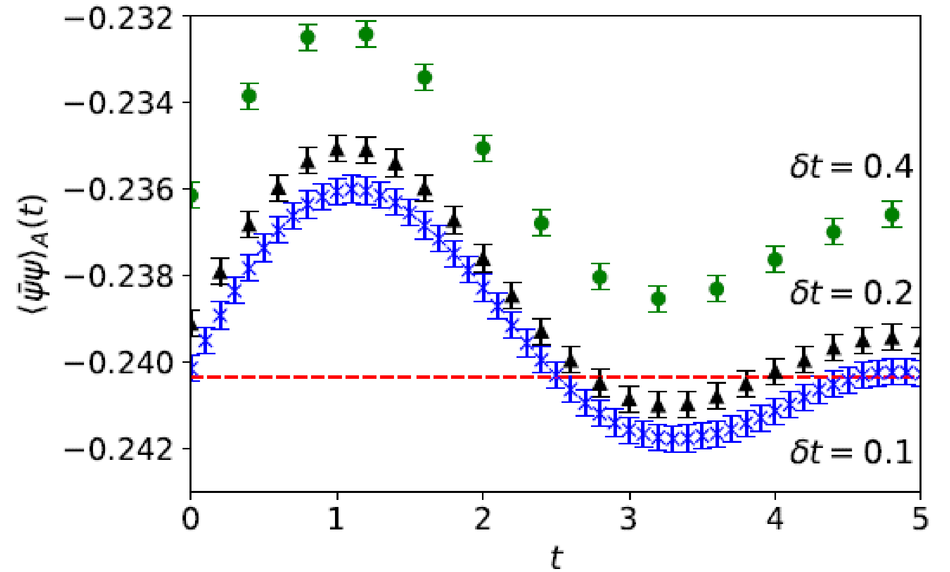
$$\begin{cases} \text{independent of t if } |\text{vac}_A\rangle = |\text{vac}\rangle \\ \text{dependent on t if } |\text{vac}_A\rangle \neq |\text{vac}\rangle \end{cases}$$

This quantity describes intrinsic ambiguities in prediction

→ Useful to estimate systematic errors

Estimation of systematic errors (Cont'd)





Oscillating around the correct value

➡ Define central value & error as

$$\frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t) + \min\langle\mathcal{O}\rangle_A(t)\right) \quad \& \quad \frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t) - \min\langle\mathcal{O}\rangle_A(t)\right)$$

Massive case

Result of mass perturbation theory: [Adam '98]

$$\langle\, \bar{\psi}(x)\psi(x)\,\rangle \simeq -0.160g + 0.322m\cos\theta + \mathcal{O}(m^2)$$

However,

$\exists$ subtlety in comparison: this quantity is UV divergent

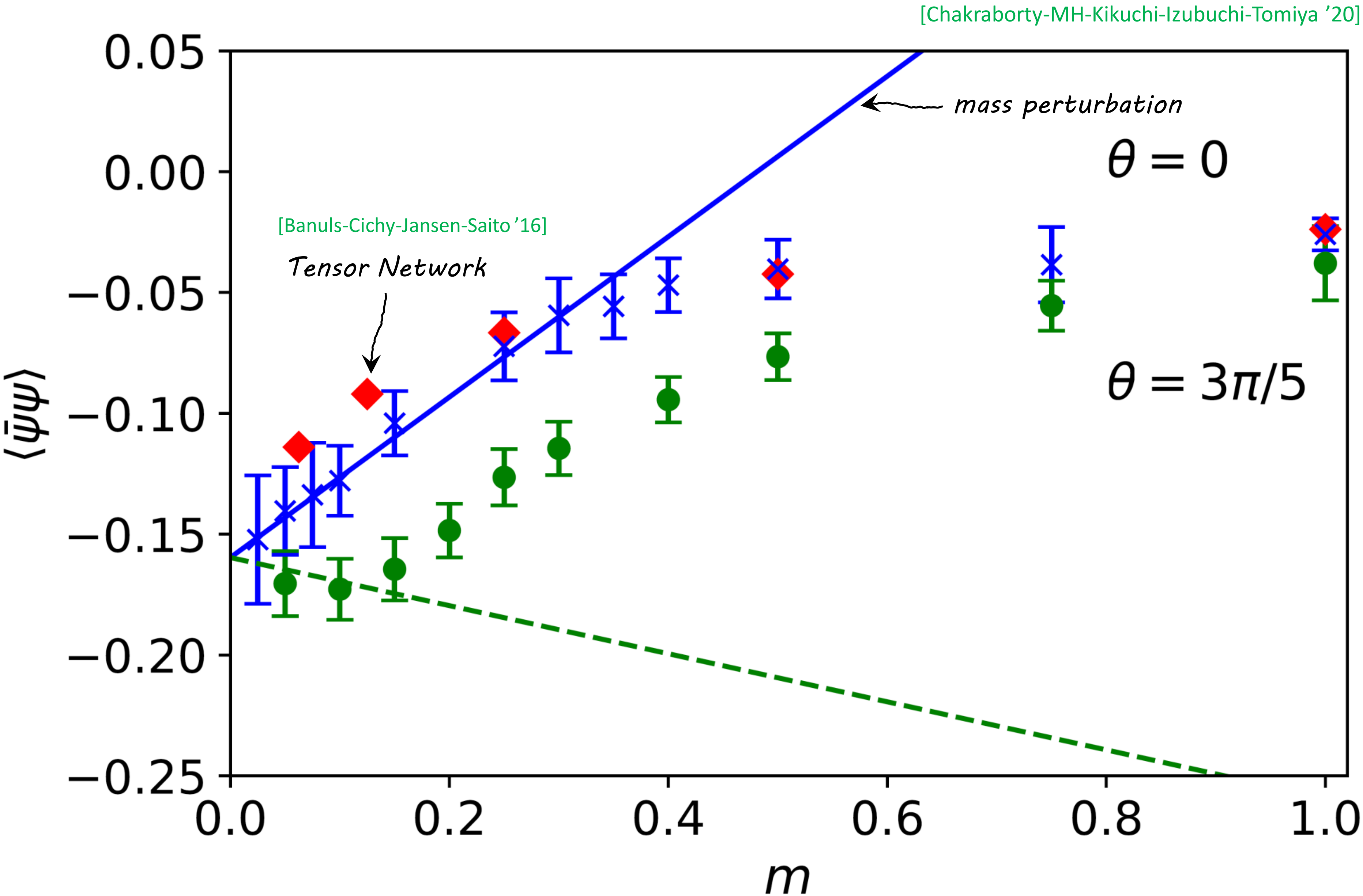
$(\sim m \log \Lambda)$

Use a regularization scheme to have the same finite part

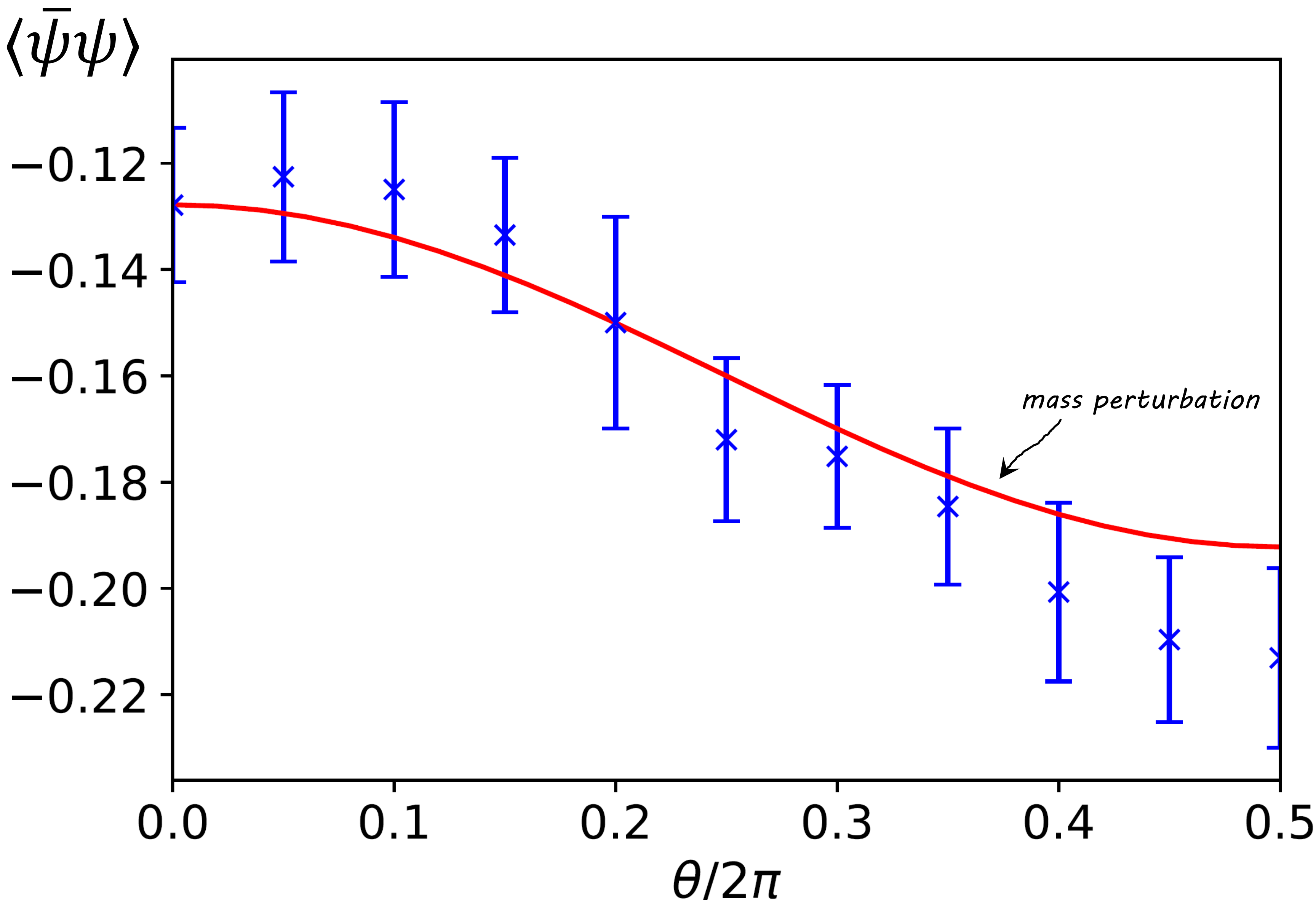
Here we subtract free theory result before taking continuum limit:

$$\lim_{a\to 0}\left[\langle\bar{\psi}\psi\rangle - \langle\bar{\psi}\psi\rangle_{\text{free}}\right]$$

Chiral condens. for massive case at g=1



θ dependence at $m = 0.1$ & $g = 1$



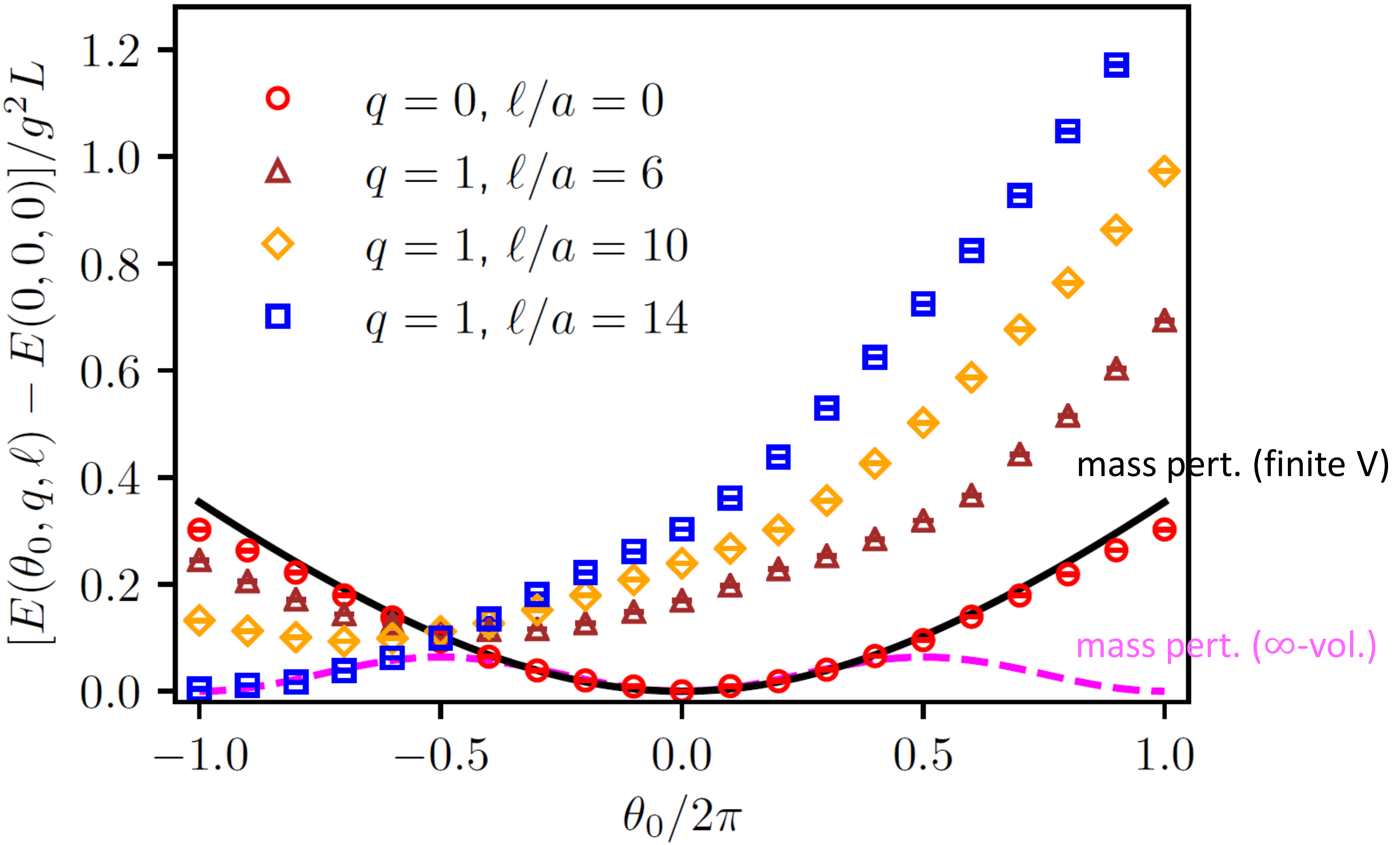
With probes

Results for $\theta_0 \neq 0$

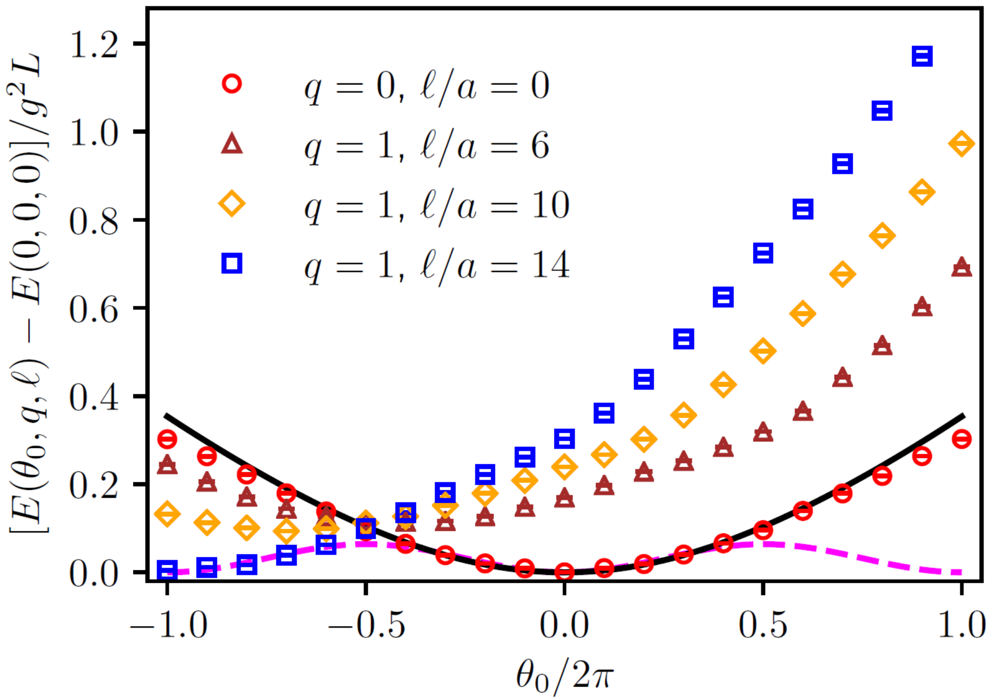
[MH-Itou-Kikuchi-Nagano-Okuda '21]

(difficult to explore by the conventional Monte Carlo approach)

Parameters: $g = 1, a = 0.4, N = 15, T = 99, q_p/q = 1, m/g = 0.2$



Comment on theta angle periodicity



Absence of the periodicity: $\theta_0 \sim \theta_0 + 2\pi$?

This is expected because we're taking open b.c.

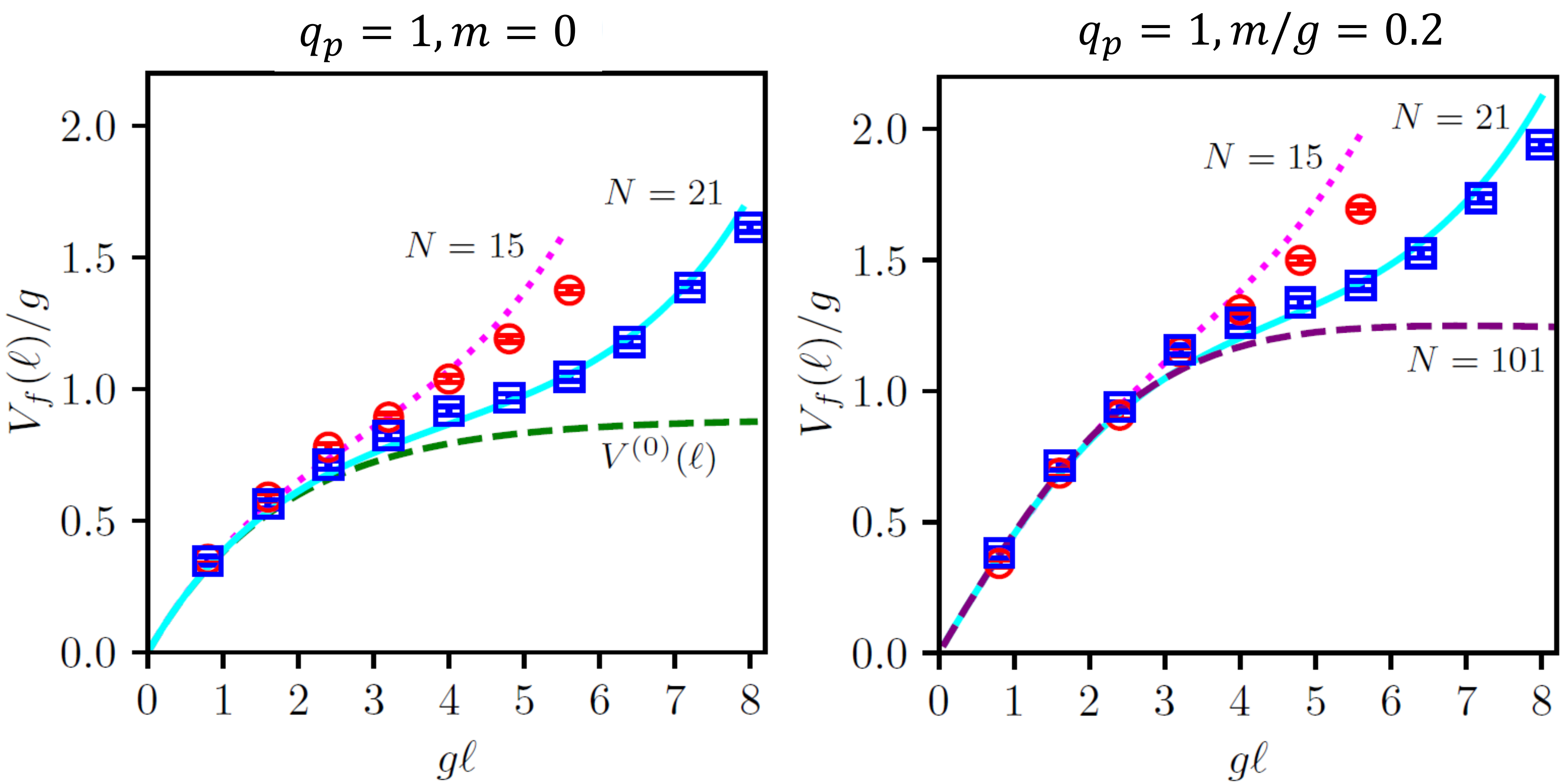
To get the periodicity back, we need to take ∞-vol. limit

Massless vs massive for $\theta_0 = 0$ & $q_p/q \in \boldsymbol{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15$ & $21, T = 99, q_p/q = 1$

Lines: analytical results in the continuum limit (finite & ∞ vols.)



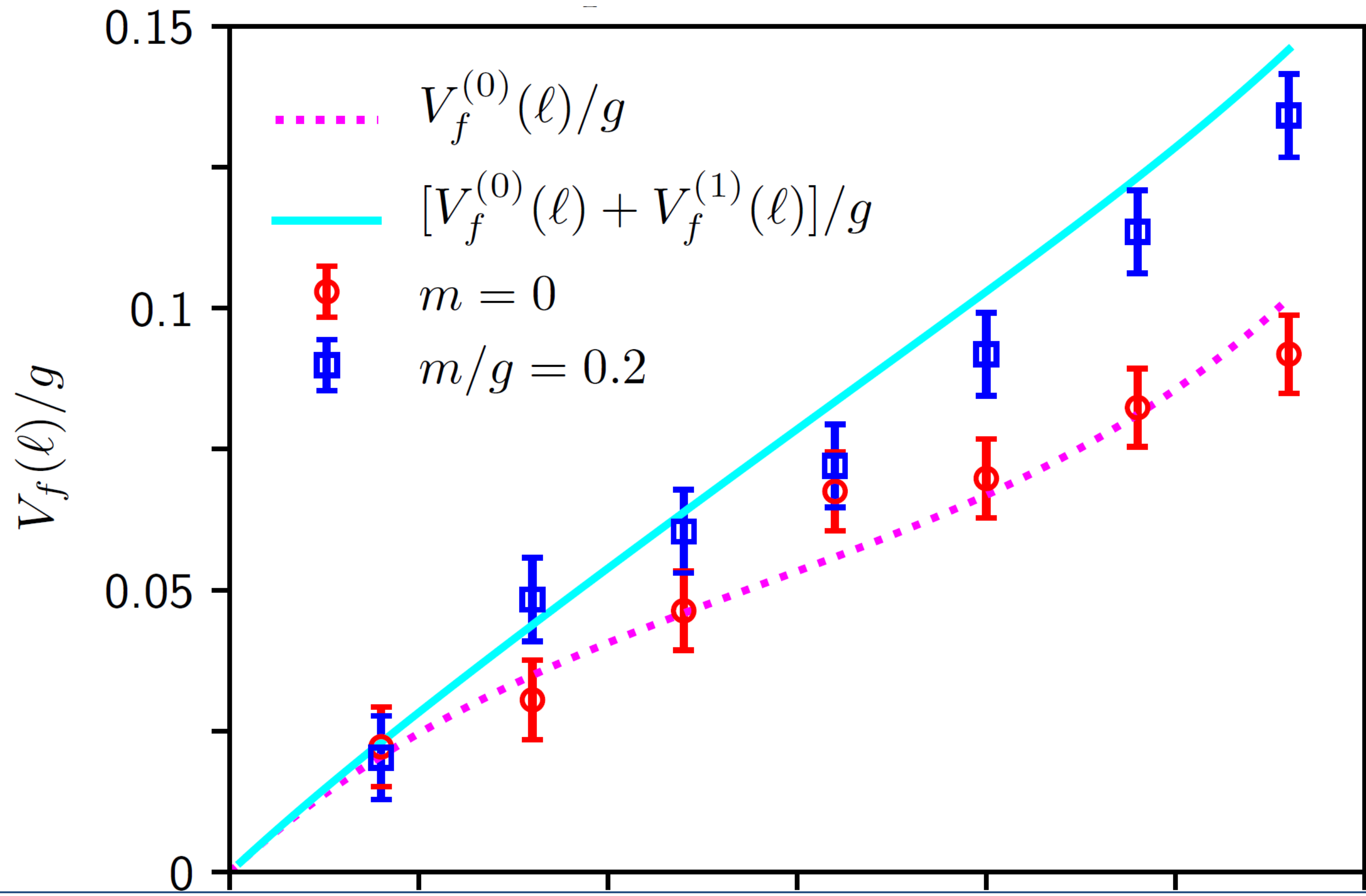
Consistent w/ expected screening behavior

Results for $\theta_0 = 0$ & $q_p/q \notin \mathbf{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15, T = 99, q_p/q = 1/4, m = 0$ & 0.2

Lines: analytical results in the continuum limit (finite & ∞ vol.)



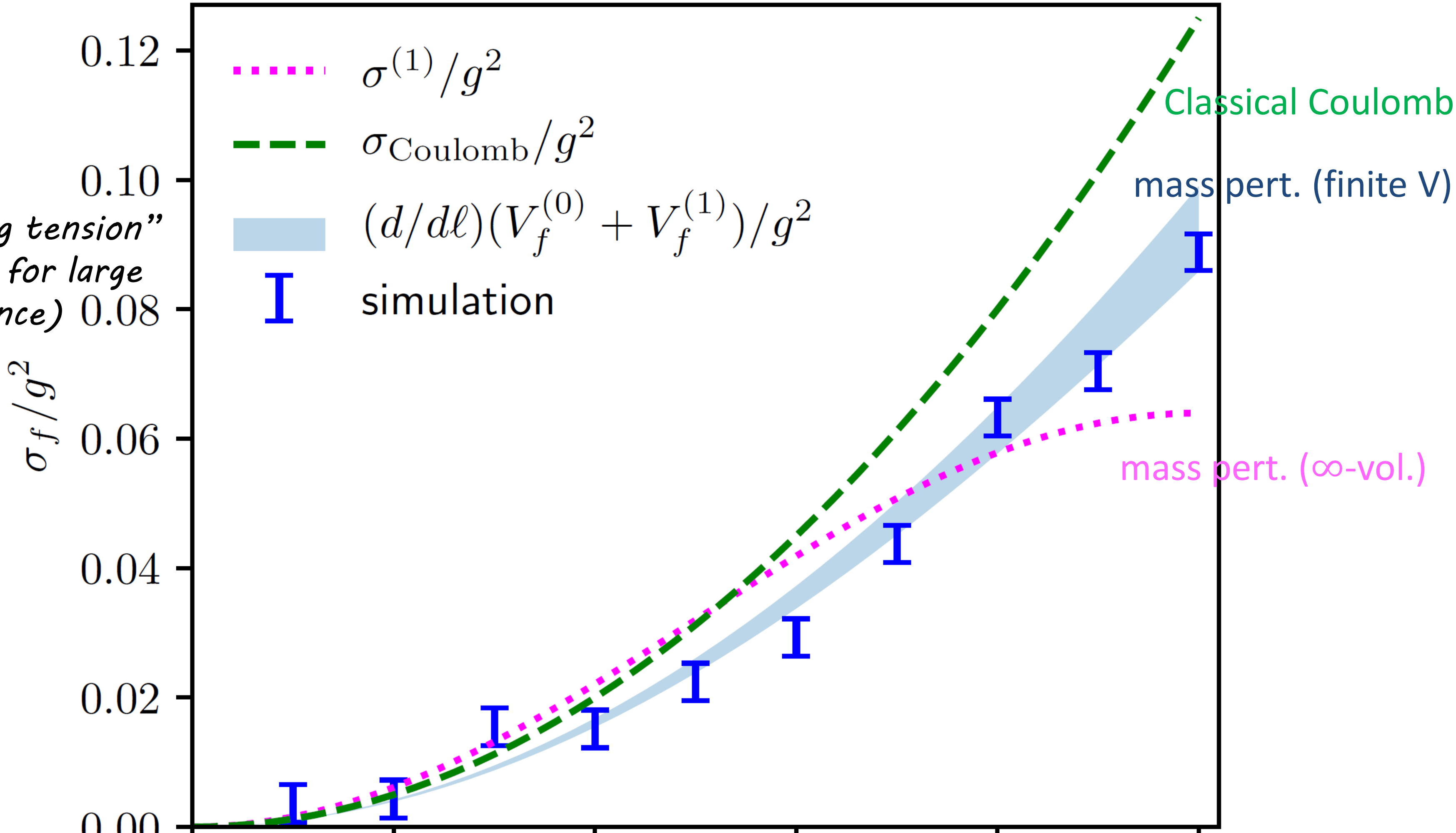
Consistent w/ expected confinement behavior

"String tension" for $\theta_0 = 0$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15, T = 99, m/g = 0.2$

"string tension" (slope for large distance)

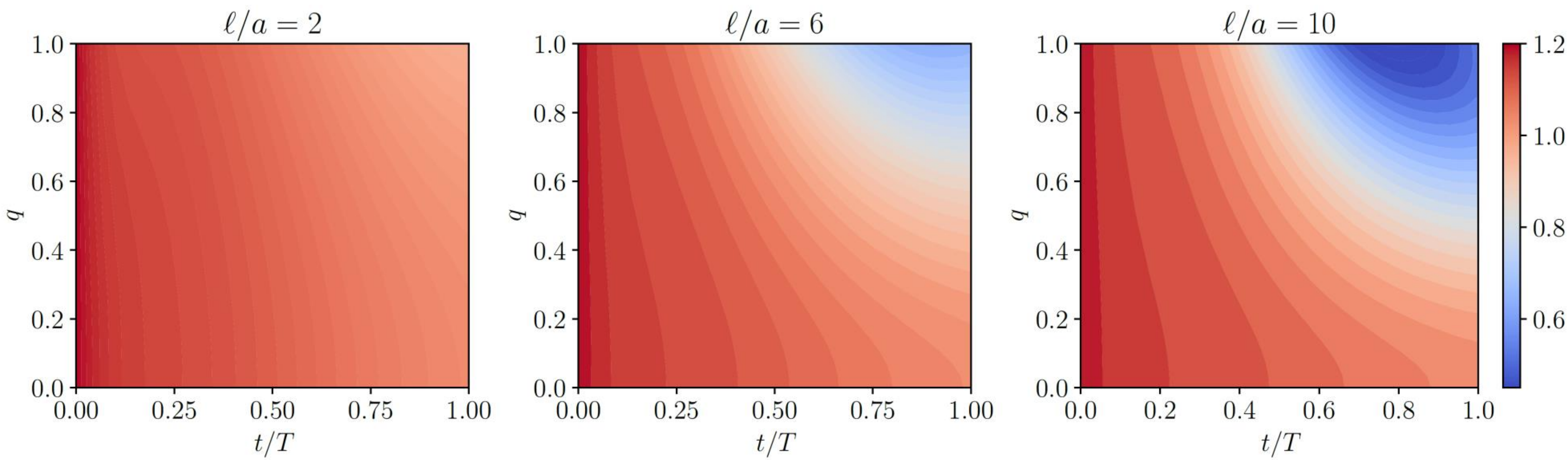


confinement by nontrivial dynamics!

Comment: density plots of energy gap

(known as "Tuna slice plot" inside the collaboration) [MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15, q_p/q = 1, m/g = 0.15$

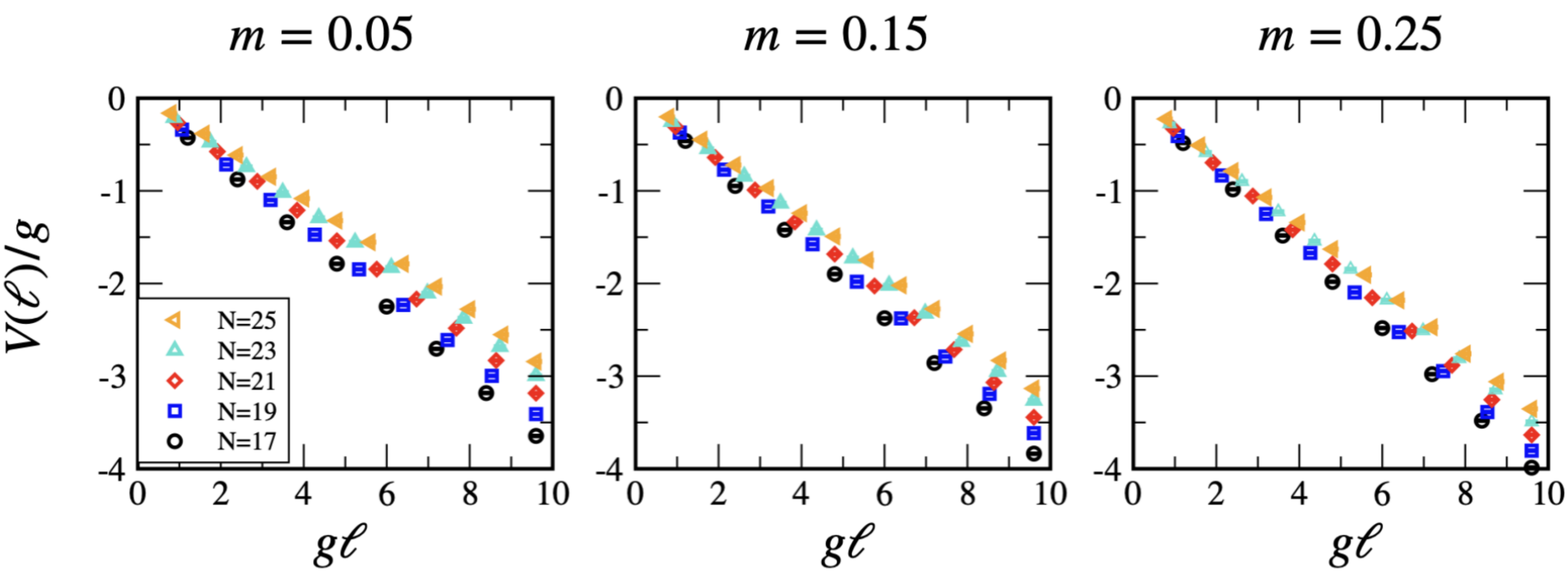


smaller gap for larger ℓ

$\Rightarrow$ larger systematic error for larger ℓ

N-dependence of V w/ fixed physical volume

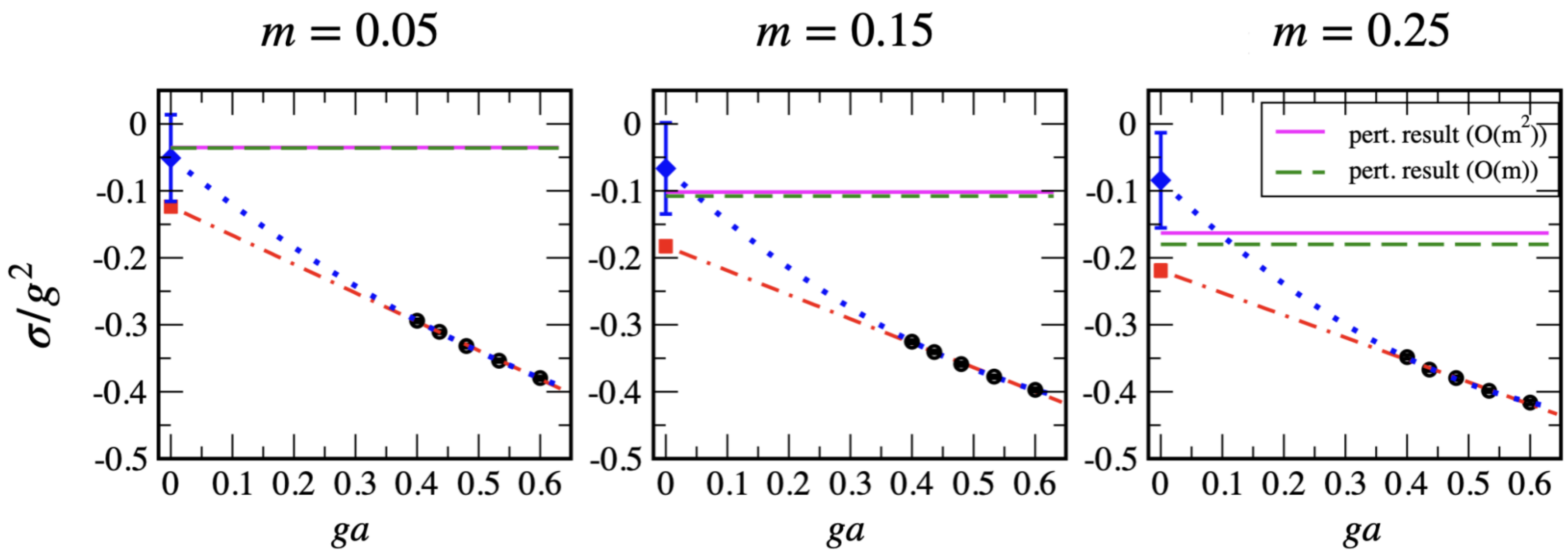
[MH-Itou-Kikuchi-Tanizaki '21]



Continuum limit of string tension

[MH-Itou-Kikuchi-Tanizaki '21]

$$g = 1,\ (\mathrm{Vol.}) = 9.6/g, T = 99,\ q_p/q = -1/3, m = 0.15, \theta_0 = 2\pi$$

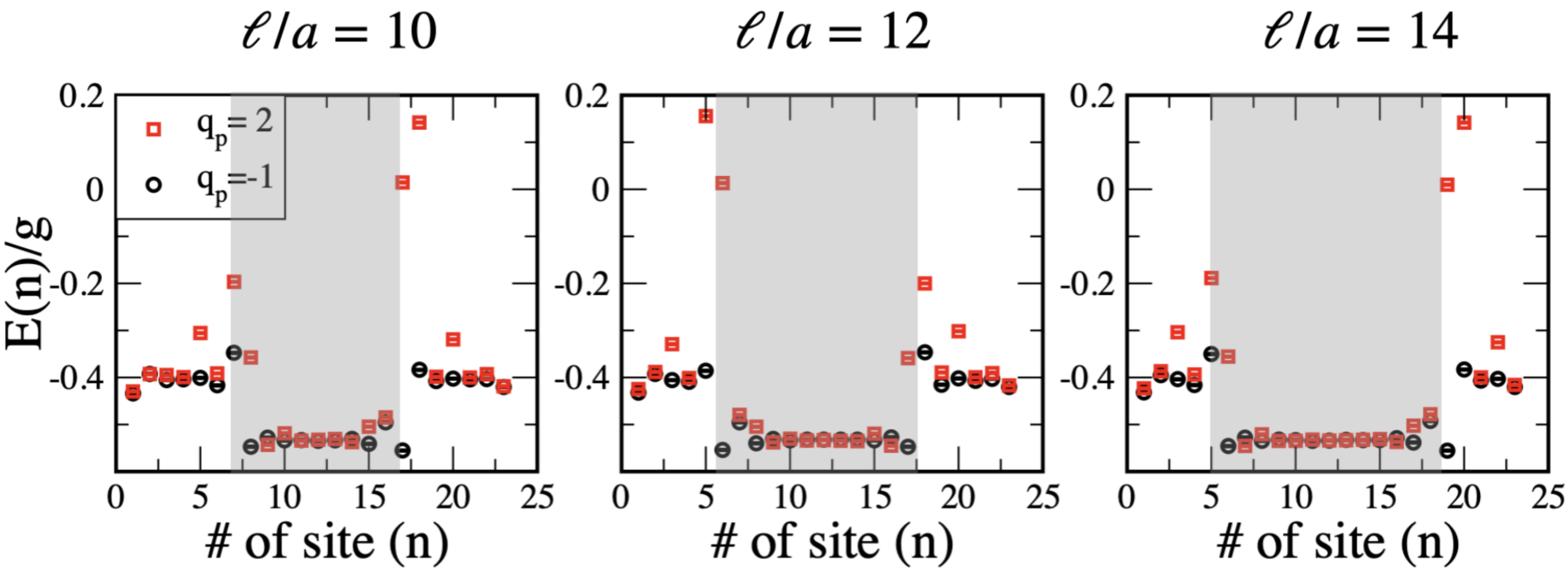


basically agrees with mass perturbation theory

Energy density @ negative tension regime

[MH-Itou-Kikuchi-Tanizaki '21]

$$g = 1, a = 0.4, N = 25, T = 99,\ q_p/q = -1/3, m = 0.15, \theta_0 = 2\pi$$

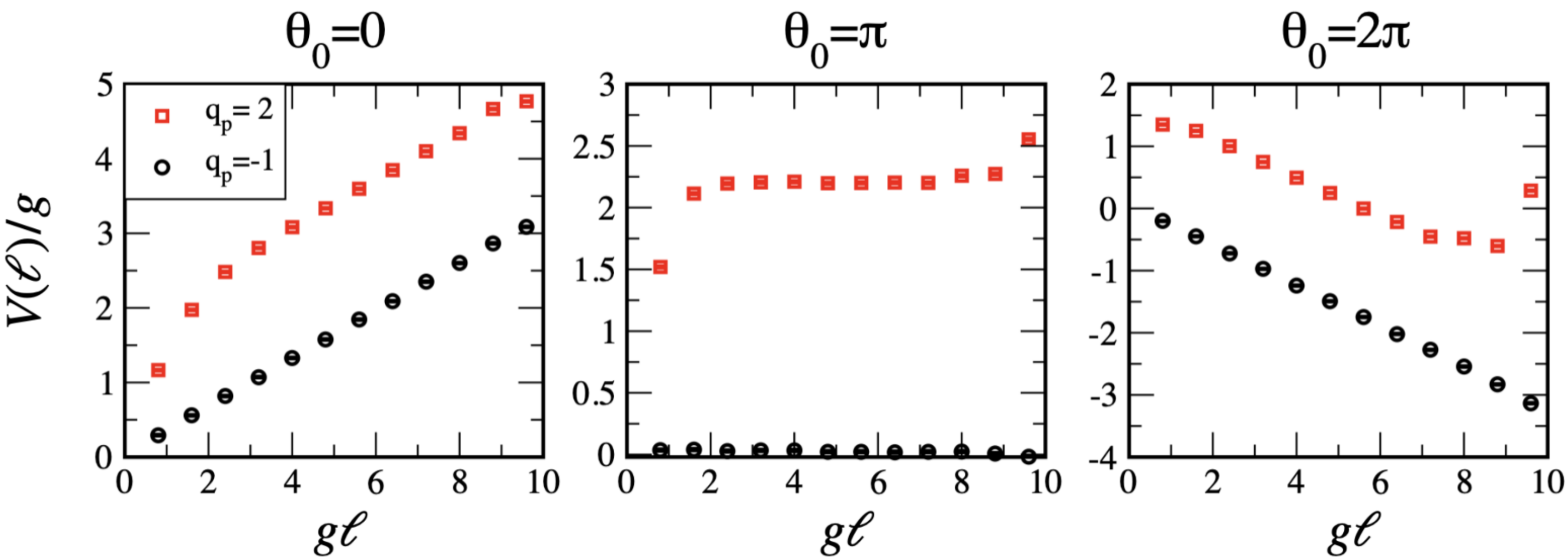


Lower energy inside the probes!!

Comparison of $q_p/q = -1/3$ & $q_p/q = 2/3$

[MH-Itou-Kikuchi-Tanizaki '21]

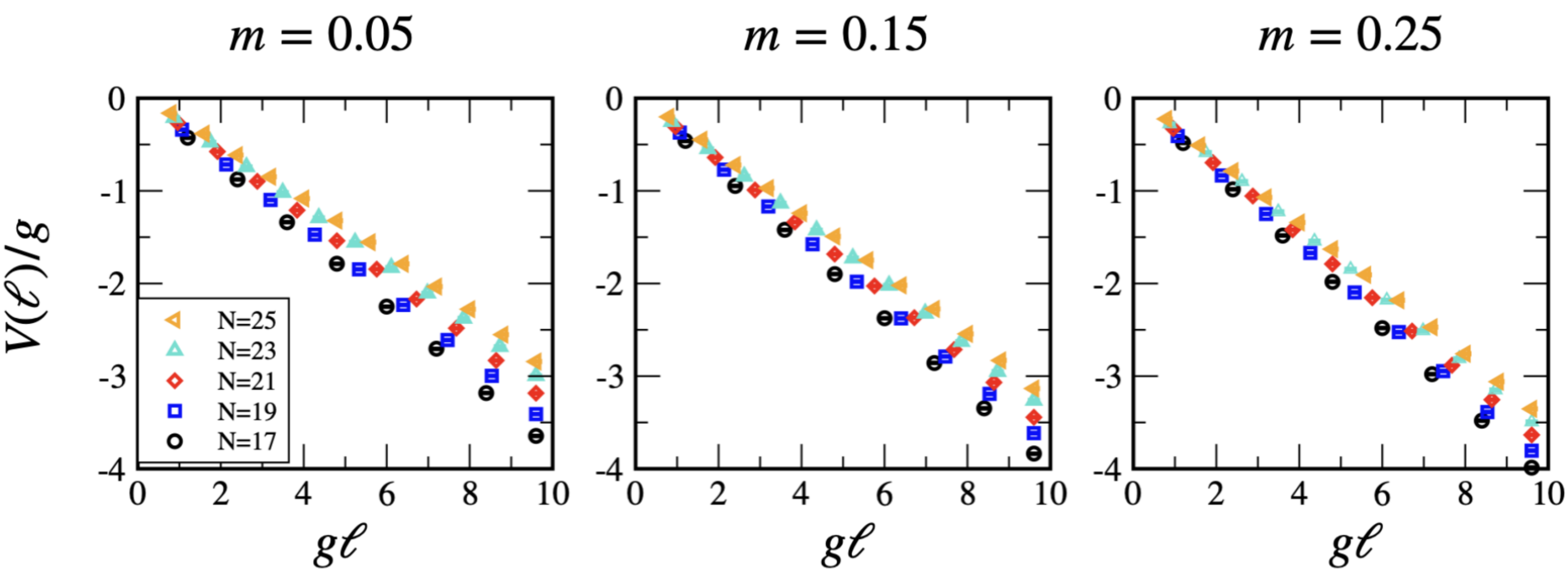
Parameters: $\mathrm{q} = 3, g = 1, a = 0.4, N = 25, T = 99, m = 0.15$



Similar slopes → (approximate) Z_3 symmetry

N-dependence of V w/ fixed physical volume

[MH-Itou-Kikuchi-Tanizaki '21]



Adiabatic scheduling

[MH-Itou-Kikuchi-Tanizaki ’21]

$$N = 17, ga = 0.40, m = 0.20, q_p = 2, \theta_0 = 2\pi,$$