Phase structure analysis of 2d CP(1) model with $\boldsymbol{\theta}$ term by TRG and CFT

H. Aizawa,

Kanazawa.U

Collaborator S.Takeda, Y.Yoshimura

2d CP(1) model

Toy model of 4d QCD

Common properties : Asymptotic freedom, confinement, θ terms, etc.

2d CP(1) model with θ terms

Action in continum

$$S=\int d^2x\left(\frac{1}{g^2}|D_\mu z(x)|^2+\frac{i\theta}{2\pi}\varepsilon_{\mu\nu}\partial_\mu A_\nu\right)$$

U(1) gauge field

complex scalar

$$z(x)=\begin{pmatrix} z_1(x) \\ z_2(x) \end{pmatrix}\in\mathbb{C}^2$$

constraint

$$|z(x)|^2=1$$

On the square lattice

$$S=-2\beta\sum_{x,\mu}\left[z^\dagger(x)z(x+\hat{\mu})U_\mu(x)+z^\dagger(x+\hat{\mu})z(x)U_\mu^{-1}(x)\right]-i\frac{\theta}{2\pi}\sum_x q(x)$$

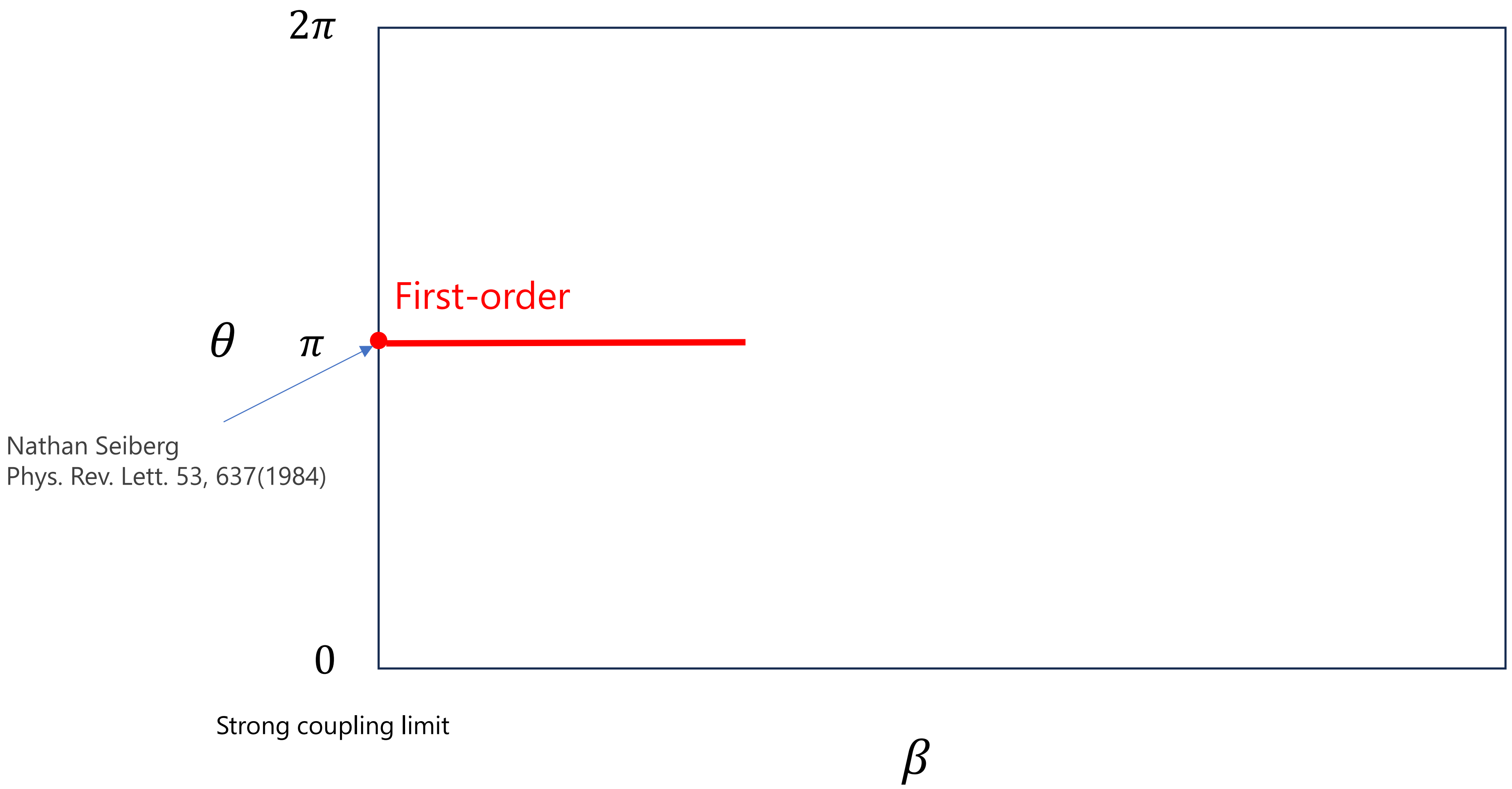
$$U_\mu(x)=e^{iA_\mu(x)}$$

$$q(x)=\frac{1}{i}\ln U_p(x)$$
$$=\{A_1(x)+A_2(x+\hat{1})-A_1(x+\hat{2})-A_2(x)\}\quad \mathrm{mod}2\pi$$

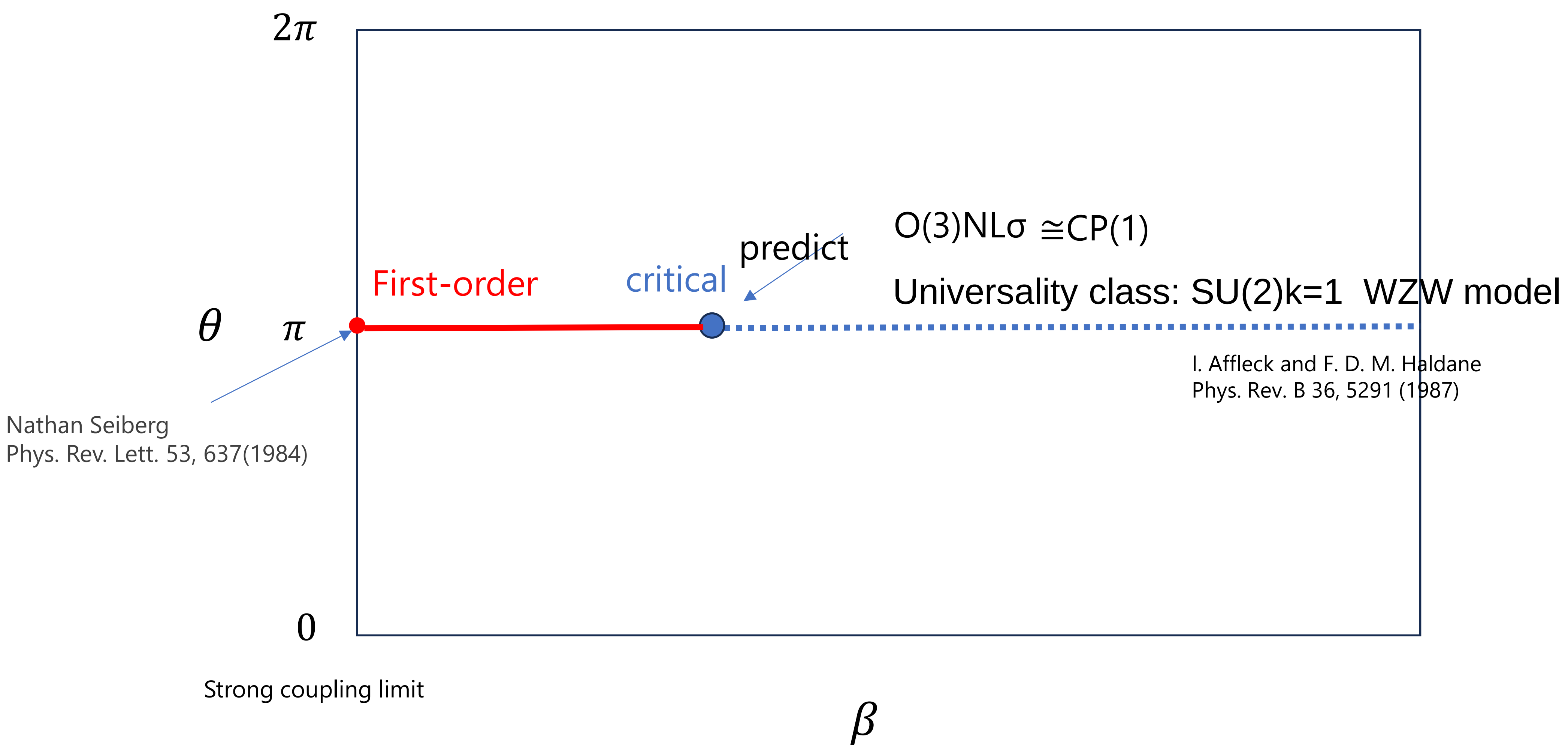
Nathan Seiberg
Phys. Rev. Lett. 53, 637

This model has sign problem

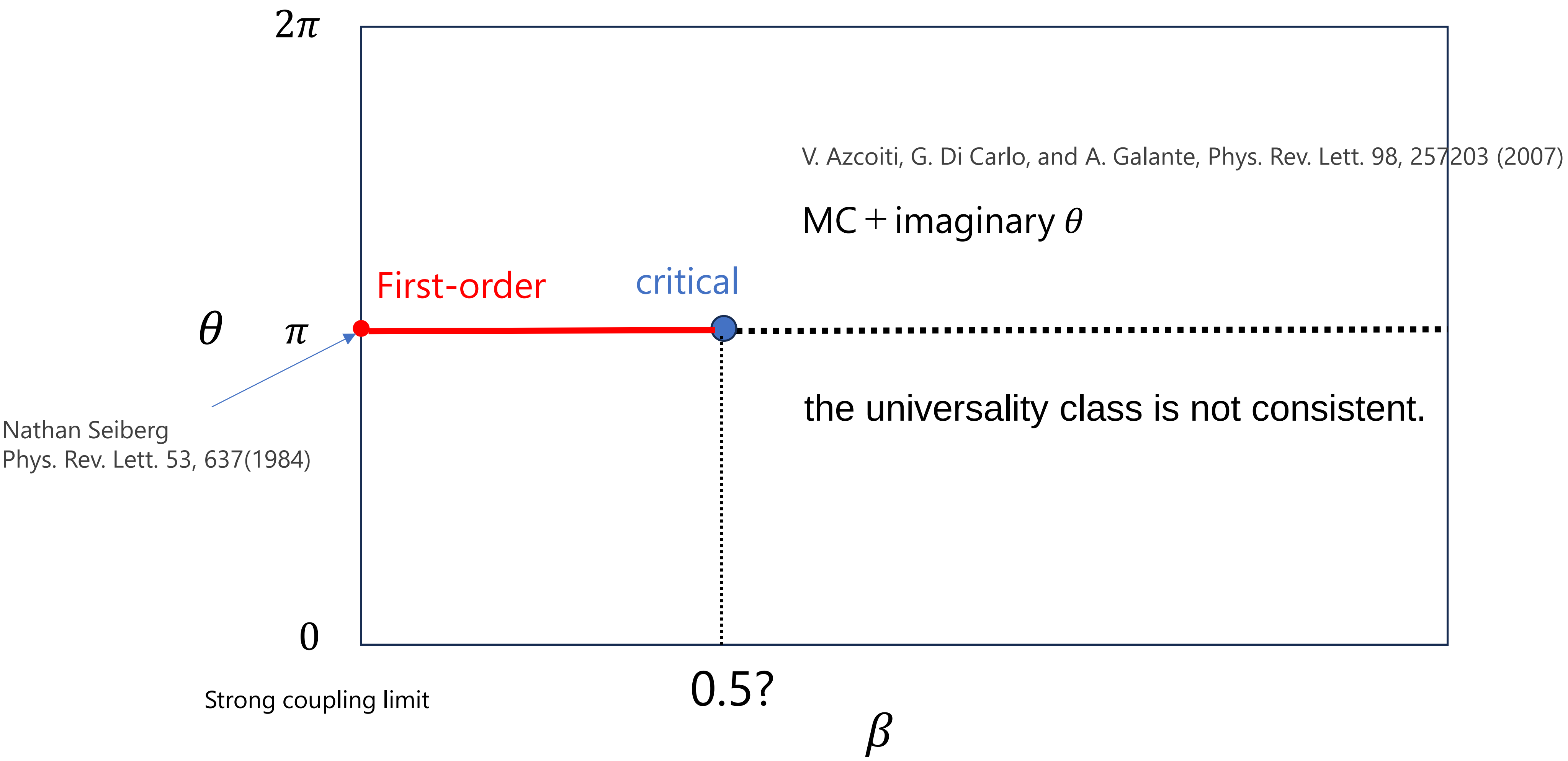
Phase structure



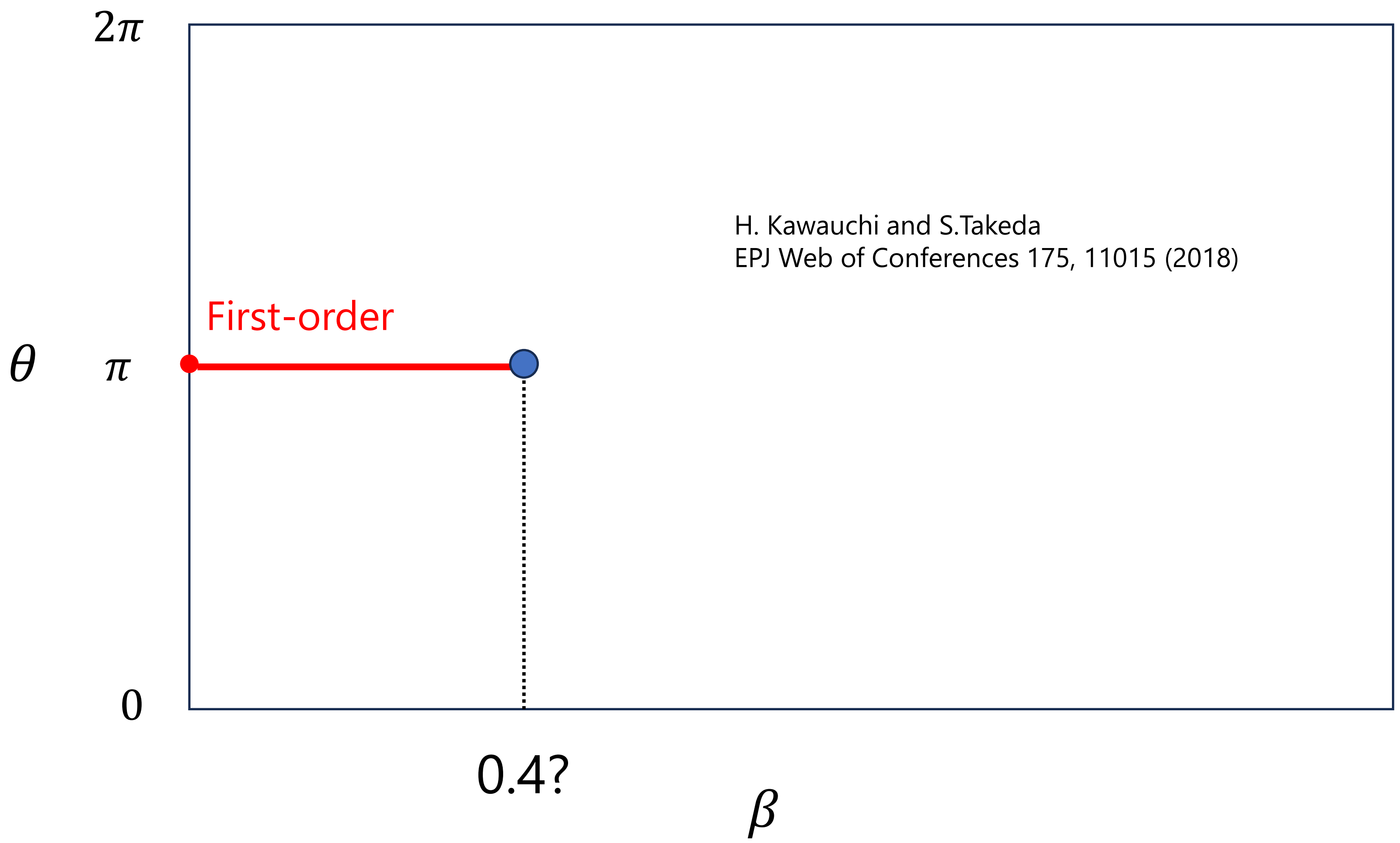
Phase structure



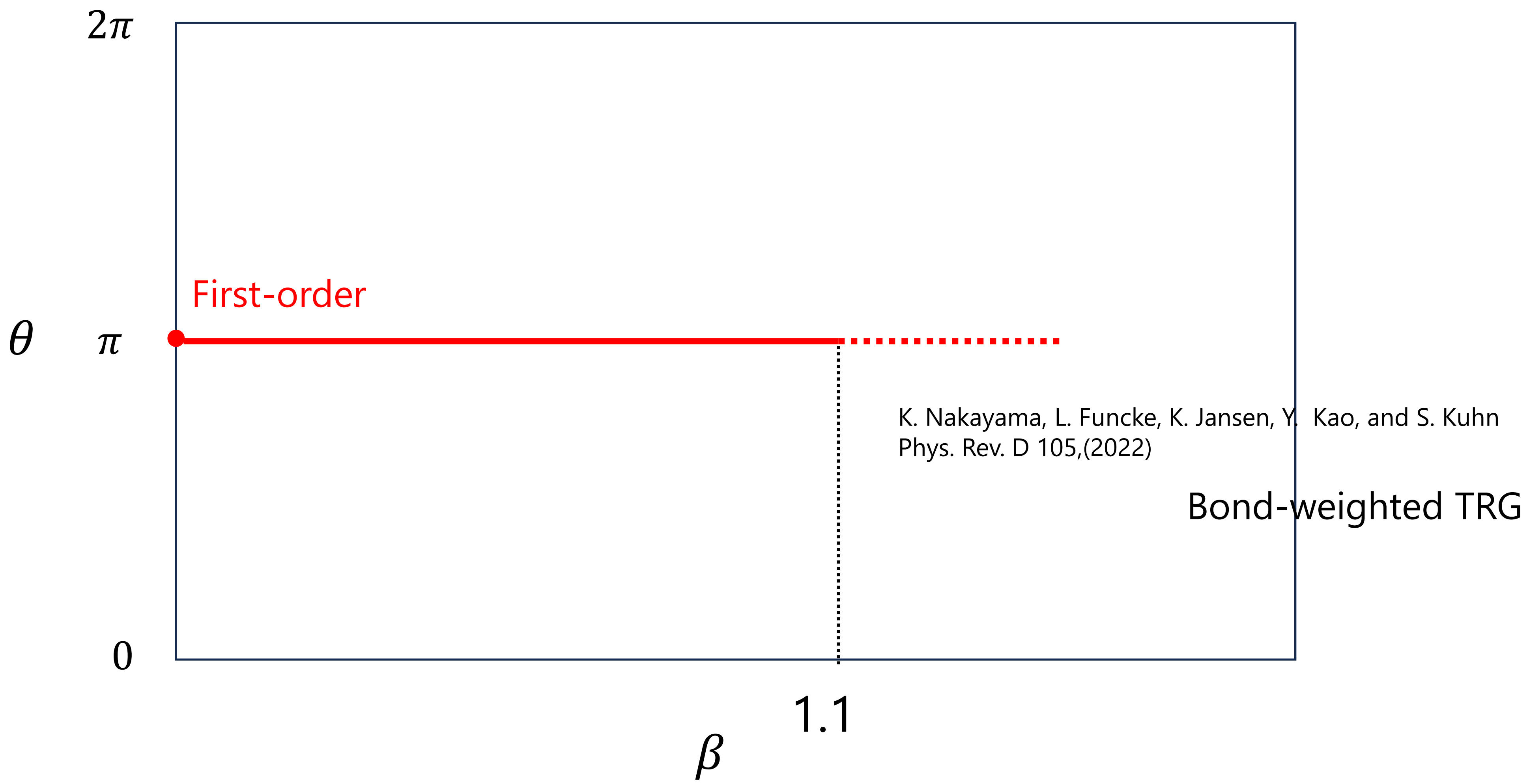
Numerical result for CP(1)



Numerical result for CP(1) using TRG



Numerical result for CP(1) using TRG



Making two improvements

- Initial tensor

- Phase structure analysis method

Initial tensor

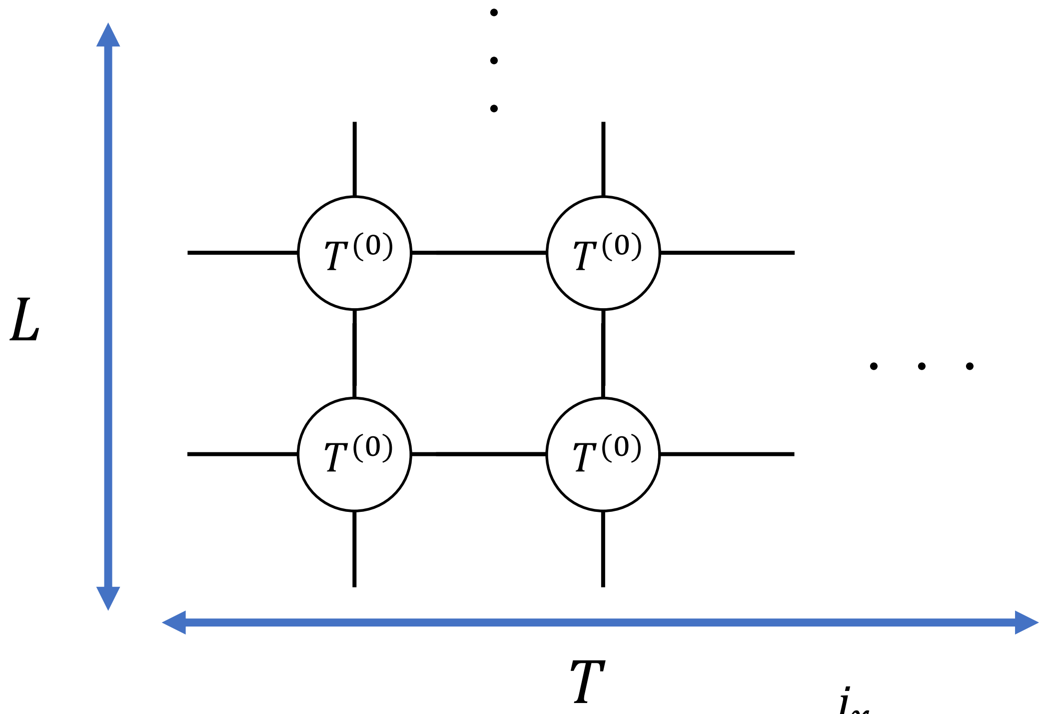
Partition function

$$Z = \int \prod_{x} dz(x) \int \prod_{x,\mu} dA_{\mu}(n) e^{-S[z,A_{\mu}]}$$

Tensor network rep.

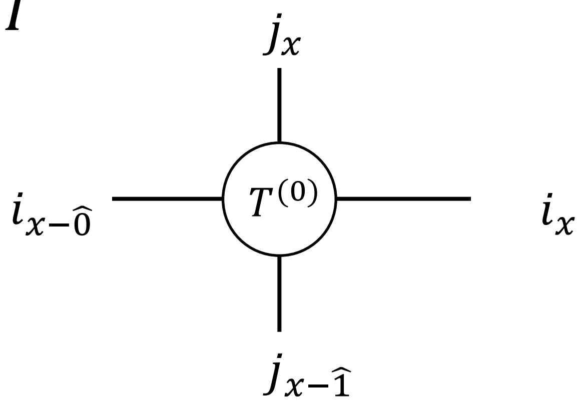
$$Z = \sum_{\{i_x, j_x\}} \prod_{x}^{N} T^{(0)}{}_{i_x, j_x, i_{x-\hat{0}}, j_{x-\hat{1}}}$$

$(N = L \times T)$



Initial tensor

$$T^{(0)}{}_{i_x, j_x, i_{x-\hat{0}}, j_{x-\hat{1}}} =$$



We need tensor that have finite index for numerical simulation

Initial tensor

Previous study

Using character expansion

$$e^{i\frac{\theta}{2\pi}q_p} = \sum_{k\in\mathbb{Z}} e^{ik(A_1+A_2-A_3-A_4)} C_k(\theta)$$

truncate

$$C_k(\theta) \propto \frac{1}{k}$$

Converge slowly

New tensor

Using quadrature

$$\int dz f(z) \approx \sum_{i=1}^{N_z} W_i^{(z)} f(z_i), \qquad \int dU f(A) \approx \sum_{a=1}^{N_A} W_a^{(A)} f(A_a)$$

→ i, a become tensor index

Scalar field Genz,Keister(1996)

Gauge field Ryo Sakai et al.(2018)

Comparison of initial tensor

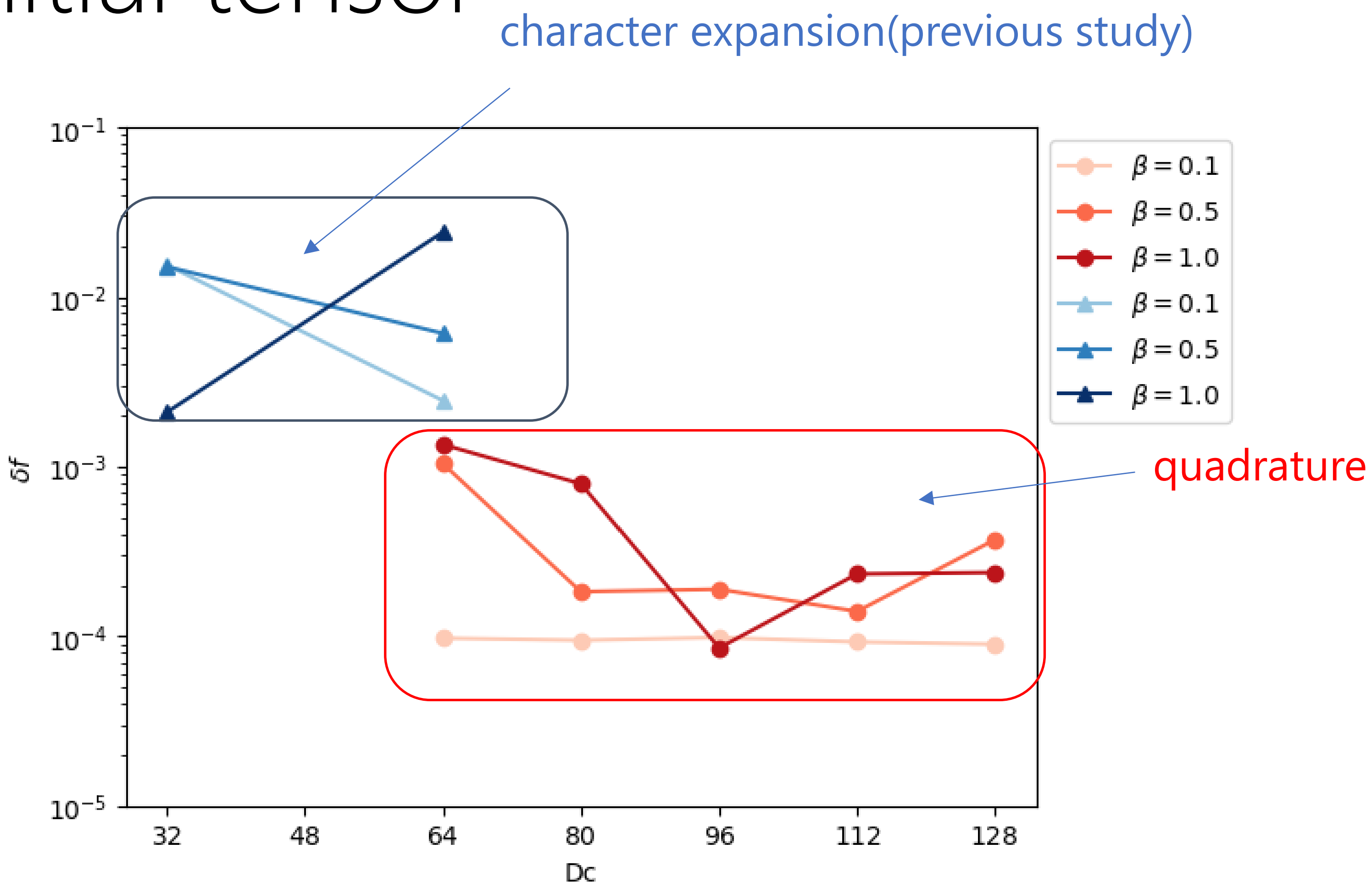
we investigate the error comparing the exact value on 2 × 2 lattice.

$$\delta f = \left| \frac{f_{tensor} - f_{exact}}{f_{exact}} \right|$$

Parameters

$N_z = 224,\ N_A = 120$

$\theta = \pi$



New initial tensor is better

Phase structure analysis method

Previous study

susceptibility $\chi = -\frac{1}{V}\frac{\partial^2 logZ}{\partial\theta^2}\Big|_{\theta=\pi}$

fitting Z near θ = π is needed

It is difficult to determine the fitting range

K. Nakayama, L. Funcke, K. Jansen, Y. Kao, and S. Kuhn
Phys. Rev. D 105,(2022)

In our study

We use central charge defined in 2d conformal field theory

Z.C. Gu and X.G. Wen
Phys. Rev. B 80, 155131 –(2009)

2d Conformal field theory

Virasoro algebra $[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}$

Algebra of 2d conformal transformation

$L_0|h\rangle = h|h\rangle \qquad L_n|h\rangle = 0,\ n > 0 \qquad n, m \in \mathbb{Z}$

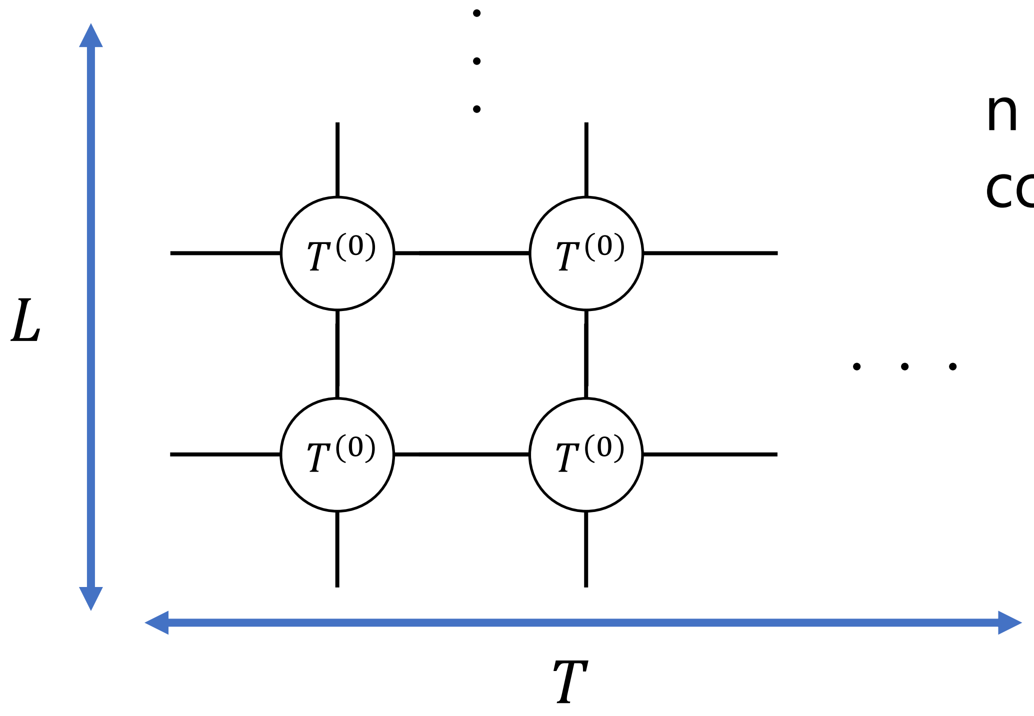
c:central charge

h:conformal weight

c and h Identify Universality

From the prediction of Haldane's conjecture,
there should be a critical point with c=1

How to compute the transfer matrix by TRG

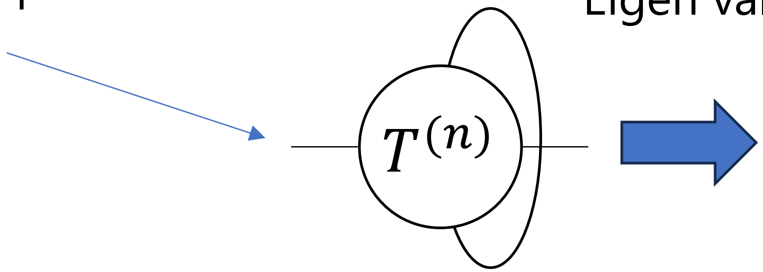


Partition function

n times
coarse-graining

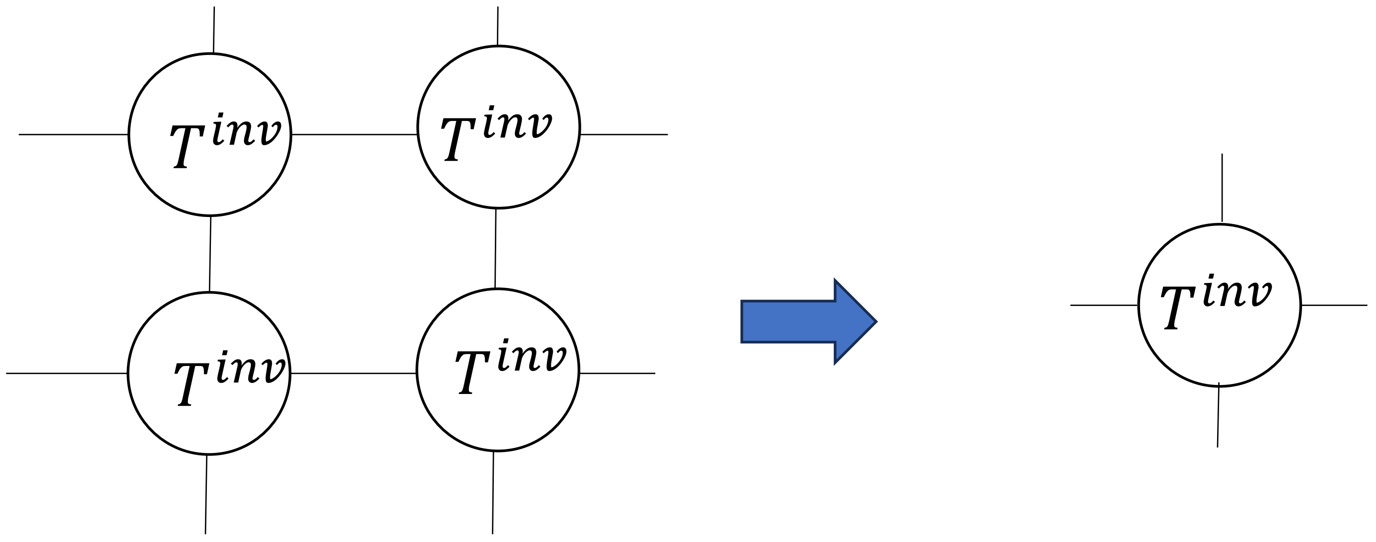
transfer matrix of a system with periodic boundary conditions in the spatial direction.

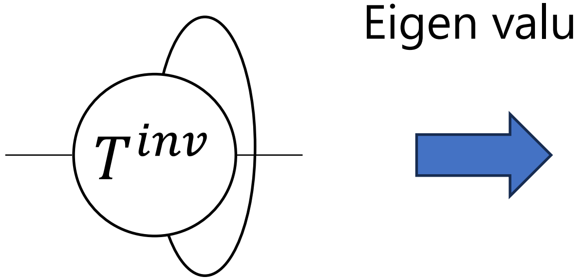
Eigen value



$$\lambda_i = e^{-E_i T}$$

When an invariant tensor is obtained under the TRG, the transfer matrix corresponds to the CFT one.





Eigen value

$$\lambda_i = e^{2\pi(h_i+\bar{h}_i)+\frac{\pi c}{6}}$$

From transfer matrix of cft

Central charge

$$c = \frac{6}{\pi}\log(\lambda_0)$$

Scaling dimension

$$x_i = h_i + \bar{h}_i = \frac{1}{2\pi}\log(\frac{\lambda_0}{\lambda_i})$$

X

Parameters

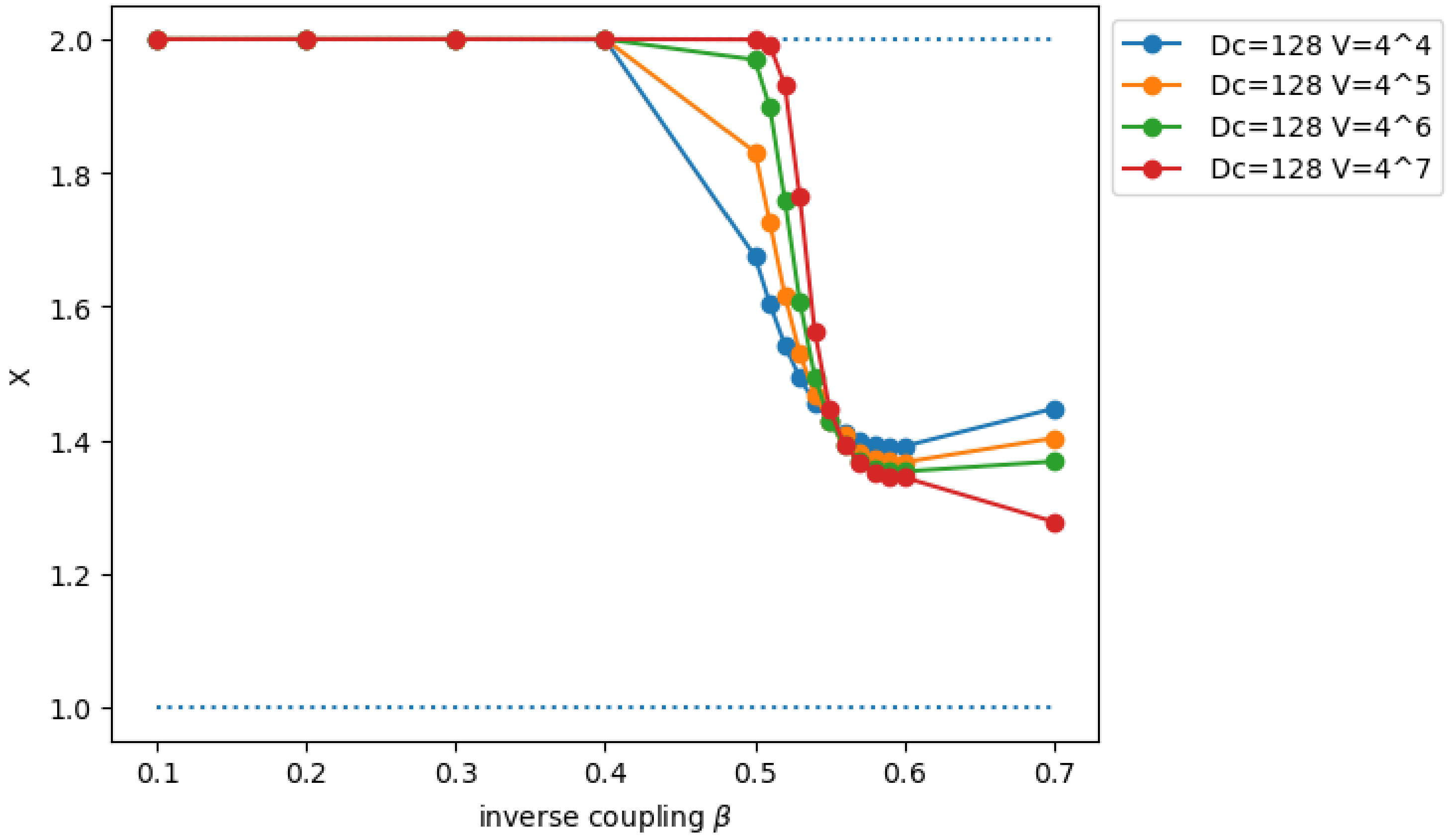
$N_z = 226, \quad N_A = 120,$
$Dc = 128, \theta = \pi$

bond-weight TRG, $\mathrm{k} = -1/2$

D. Adachi, T. Okubo, and S. Todo,
Phys. Rev. B 105, L060402(2022)

$$X = \frac{(\sum_i T_{ii})^2}{\sum_{ij} T_{ij} T_{ji}} = \frac{(\sum_i \exp[-E_i T])^2}{\sum_i \exp[-2E_i T]}$$

If $T \to \infty,\ E_0 = E_1, \quad X = 2$



We find first-order transition up to $\beta = 0.5$

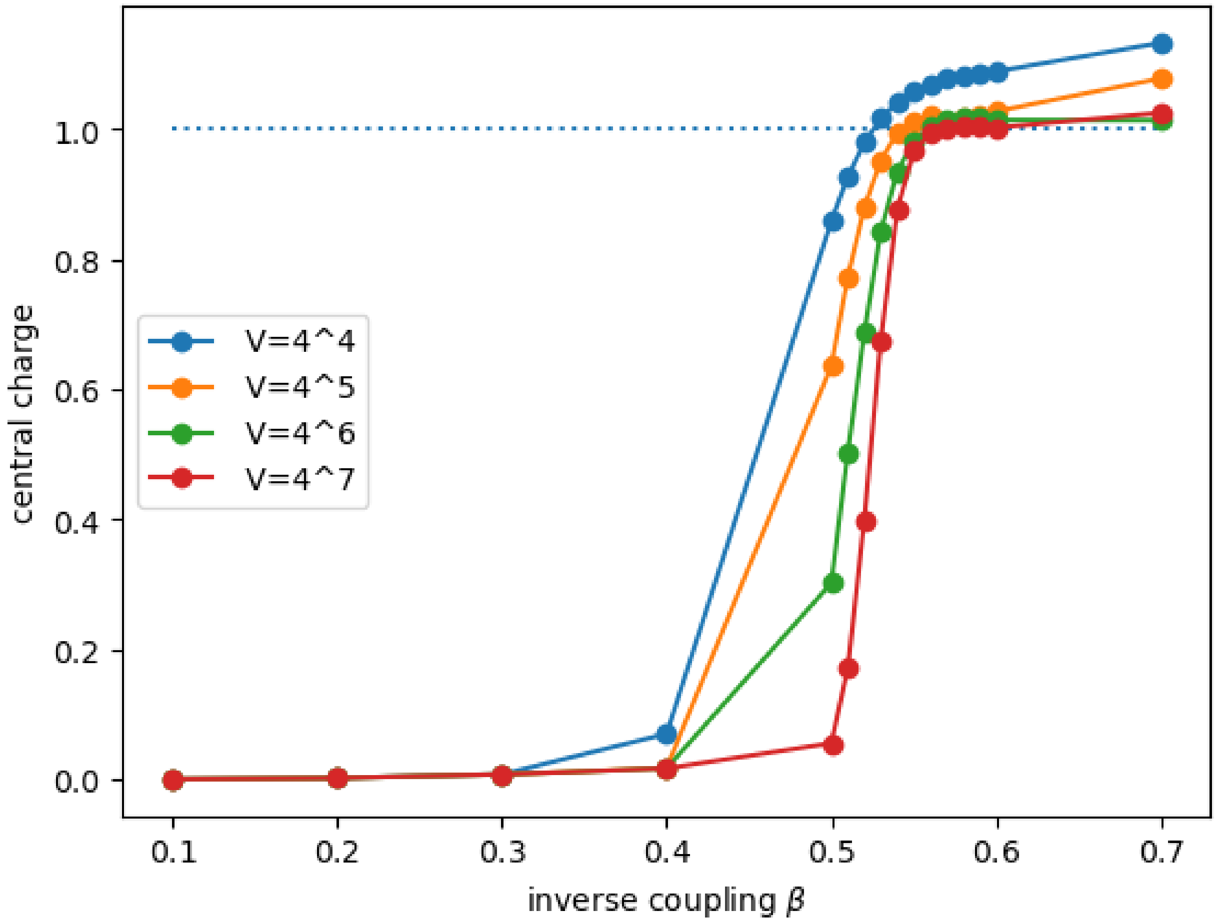
Central charge

$c = 1$ for $\beta \geq 0.55$

Parameters

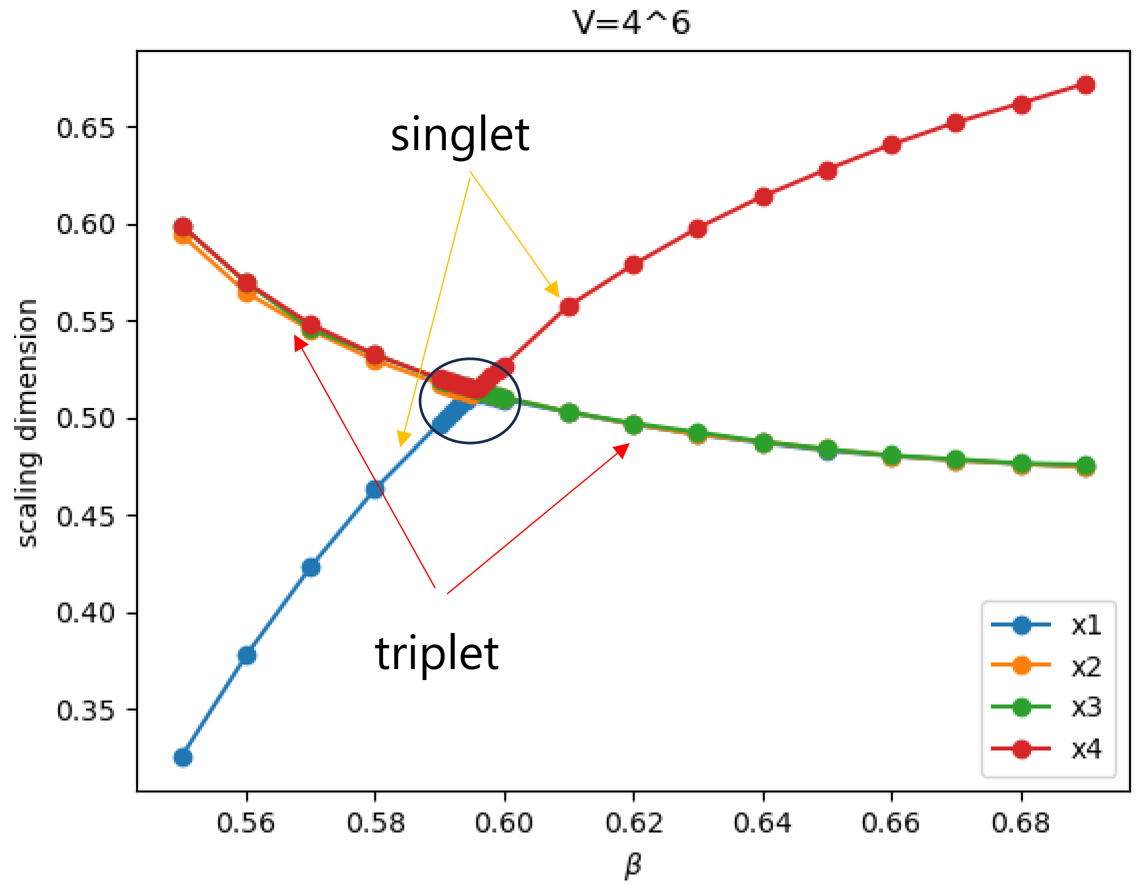
$N_z = 226, \;\; N_A = 120,$
$Dc = 128, \theta = \pi$

coarse-graining by bond-weight TRG,
$\mathrm{k} = -1/2$



Level spectroscopy

K Nomura and K Okamoto 1994
J. Phys. A: Math. Gen. 27 5773



$$x_s(L) = x_{s,c} + \alpha_s(L)(\beta - \beta_c) + \delta_s$$

$$x_t(L) = x_{t,c} + \alpha_t(L)(\beta - \beta_c) + \delta_t$$

$$x_s(L) = x_t(L), \qquad at \quad \beta = \beta^*$$

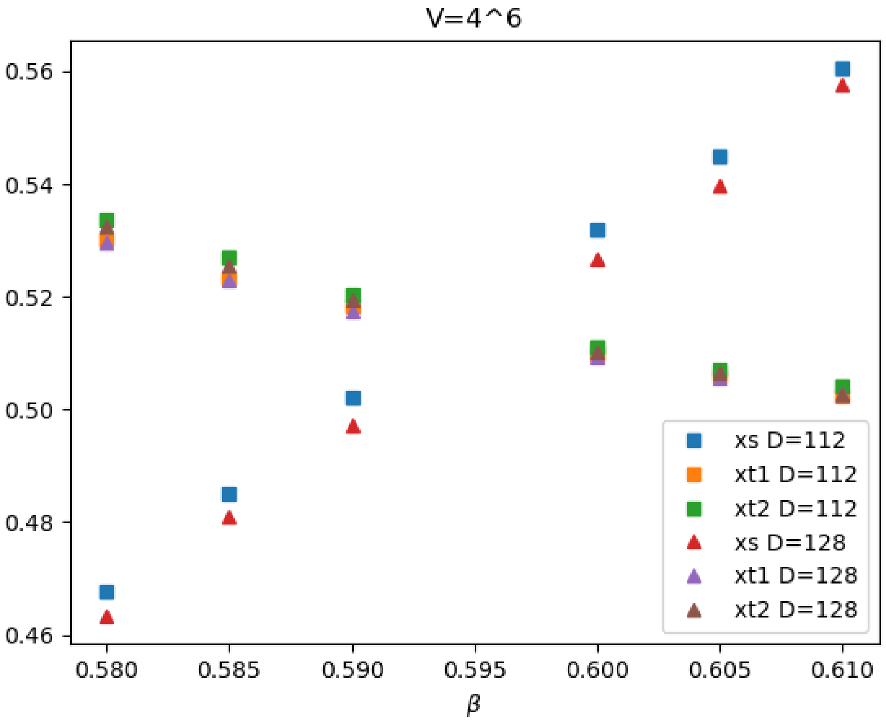
$$\beta^* = \beta_c - \frac{\delta_s - \delta_t}{\alpha_s - \alpha_t}$$

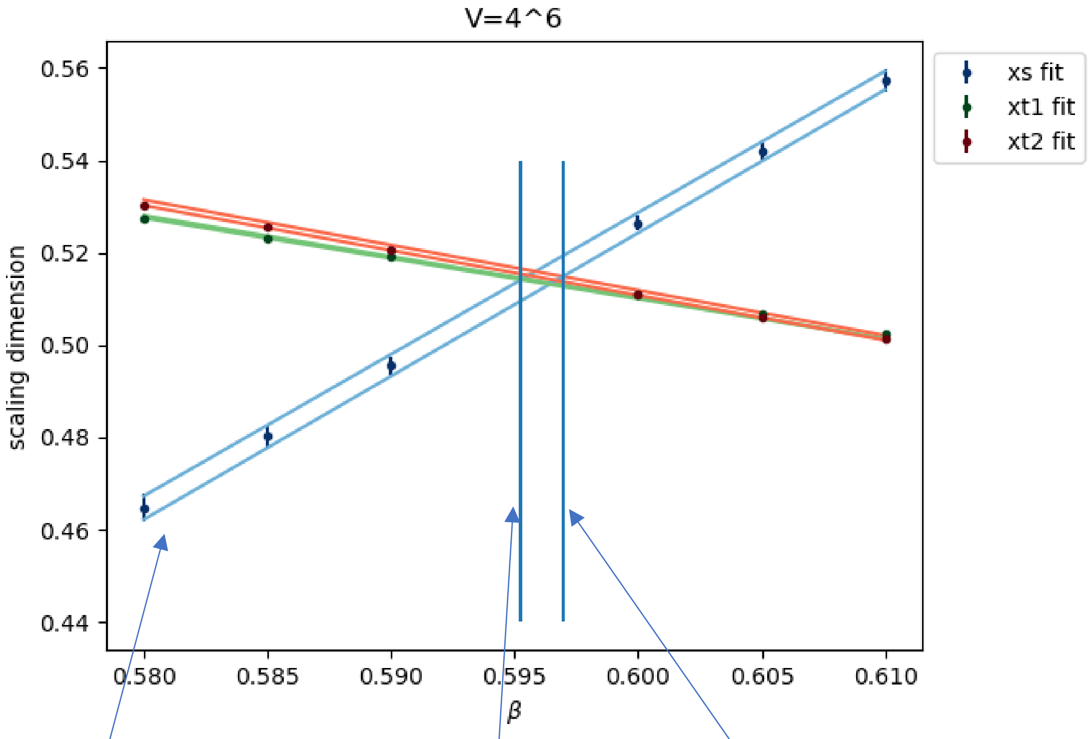
$$\delta \sim \frac{1}{L^2}$$

$$\alpha \sim \frac{1}{\log(L)}$$

Log correction
is canceled

Fitting result





β_{min} β_{max}

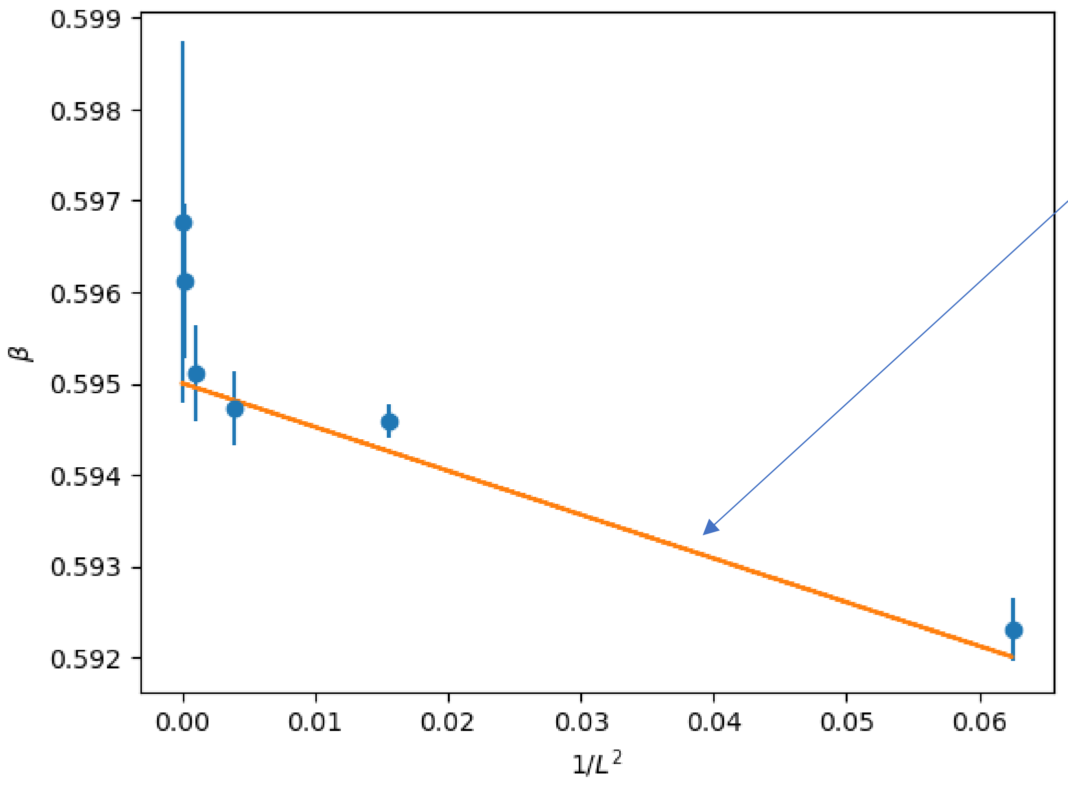
fit $$x = a(\beta - 0.598) + b$$

$x = b$ at 0.598
$xerr = berr$

$$\beta^* = \frac{\beta_{max} + \beta_{max}}{2}$$

$$\beta^* err = \frac{\beta_{max} - \beta_{max}}{2}$$

Fitting result



fit $\beta^* = \beta_c - \frac{C}{L^2}$

$$\frac{\chi^2}{N-2} = 1.7241329706647415$$

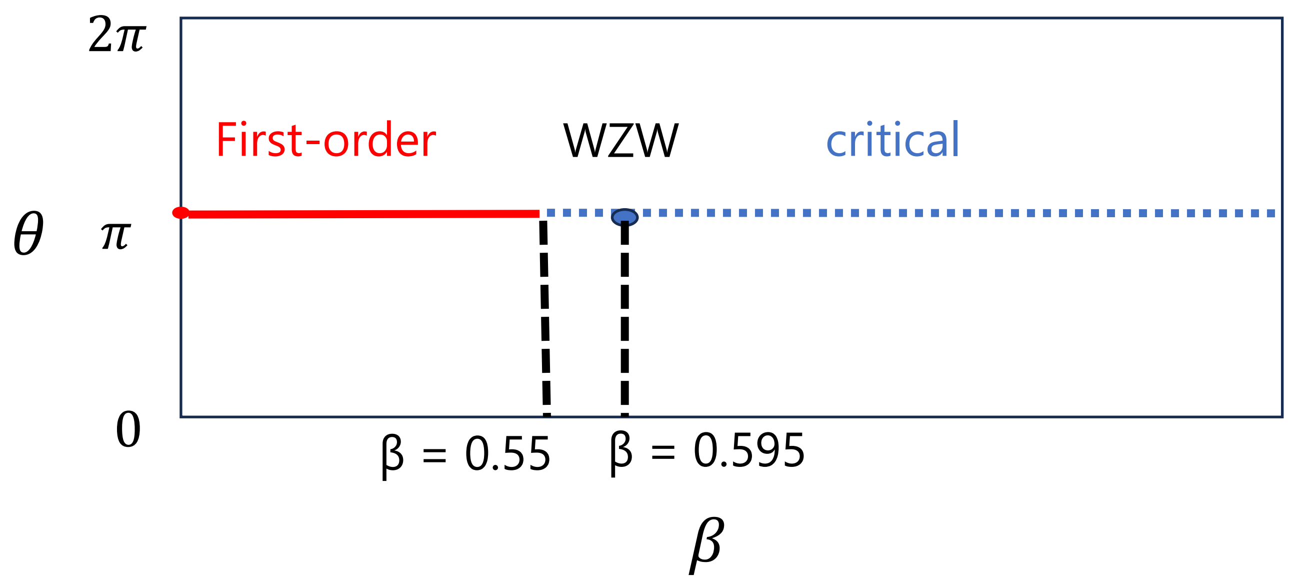
The critical point is found at

$$\beta_c = 0.5952(2)$$

Summary

Two improvements

- Initial tensor
- New analysis using CFT : central charge and scaling dimensions



The critical point predicted by Haldane's conjecture is found