Floquet prethermalization of lattice gauge theory on superconducting qubits

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Collaborators: Yoshimasa Hidaka (YITP), Yuta Kikuchi (Quantinuum), Kazuhiro Seki (RIKEN), Arata Yamamoto (U. Tokyo), Seiji Yunoki (RIKEN)

References:

Kazuhiro Seki, Yuta Kikuchi, Tomoya Hayata, Seiji Yunoki, arXiv: 2405.07613 [quant-ph]

Tomoya Hayata, Kazuhiro Seki, Arata Yamamoto, arXiv:2408.10079 [hep-lat]

Tomoya Hayata, Yoshimasa Hidaka, arXiv: 2409.20263 [hep-lat]

see also: Zache, González-Cuadra, Zoller, PRL 131 171902 (2023)

TH, Hidaka, JHEP 2023, 126 (2023); JHEP 2023, 123 (2023)

My ultimate goal

$$i\partial_t|\Psi\rangle = H|\Psi\rangle$$

Solve the real-time dynamics of QFT (QCD)

⬇

Quantum computing of quantum field theories

Two directions

○ Implementing QCD on quantum devices is never trivial

➡ Develop quantum algorithms and wait fault-tolerant QCs

➡ Simulating toy models of LGTs on NISQ devices

e.g., Schwinger model (QED in 1+1 dimensions)

Artificial tasks
(Random circuits)

Google ‘19
Zuchongzhi ’21
Quantinuum ‘24

What can we do?

Important tasks

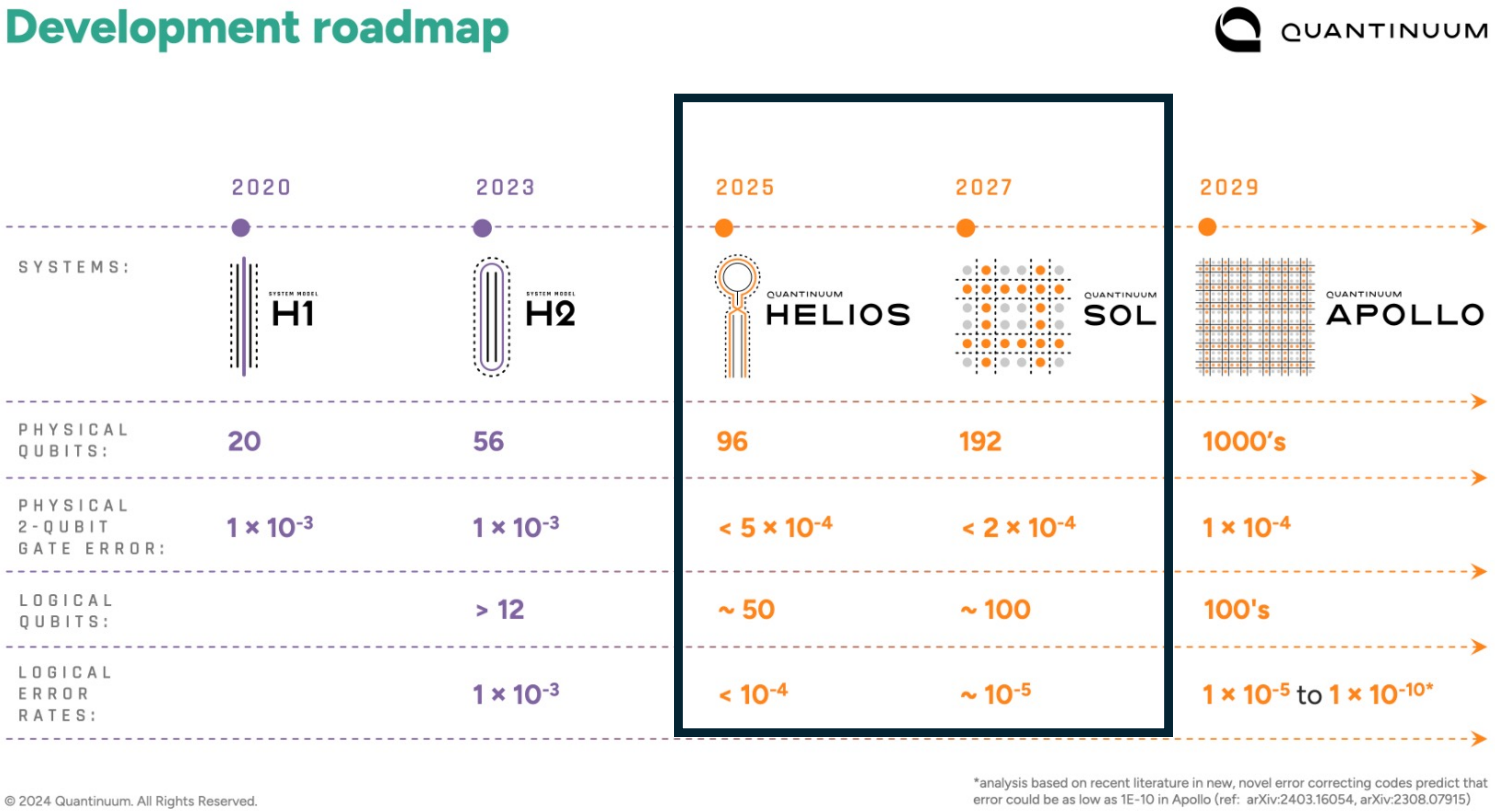
Prime factoring
Quantum chemistry
Cond-mat physics
⋮

Time

Deal with noises

Fault-tolerant QC

What can we do?



Development roadmap

QUANTINUUM

	2020	2023	2025	2027	2029
SYSTEMS:	SYSTEM MODEL H1	SYSTEM MODEL H2	QUANTINUUM HELIOS	QUANTINUUM SOL	QUANTINUUM APOLLO
PHYSICAL QUBITS:	20	56	96	192	1000's
PHYSICAL 2-QUBIT GATE ERROR:	1×10^{-3}	1×10^{-3}	$< 5 \times 10^{-4}$	$< 2 \times 10^{-4}$	1×10^{-4}
LOGICAL QUBITS:		> 12	~ 50	~ 100	100's
LOGICAL ERROR RATES:		1×10^{-3}	$< 10^{-4}$	$\sim 10^{-5}$	1×10^{-5} to 1×10^{-10}*

*analysis based on recent literature in new, novel error correcting codes predict that error could be as low as 1E-10 in Apollo (ref: arXiv:2403.16054, arXiv:2308.07915)

physical qubits are more useful than logical qubits

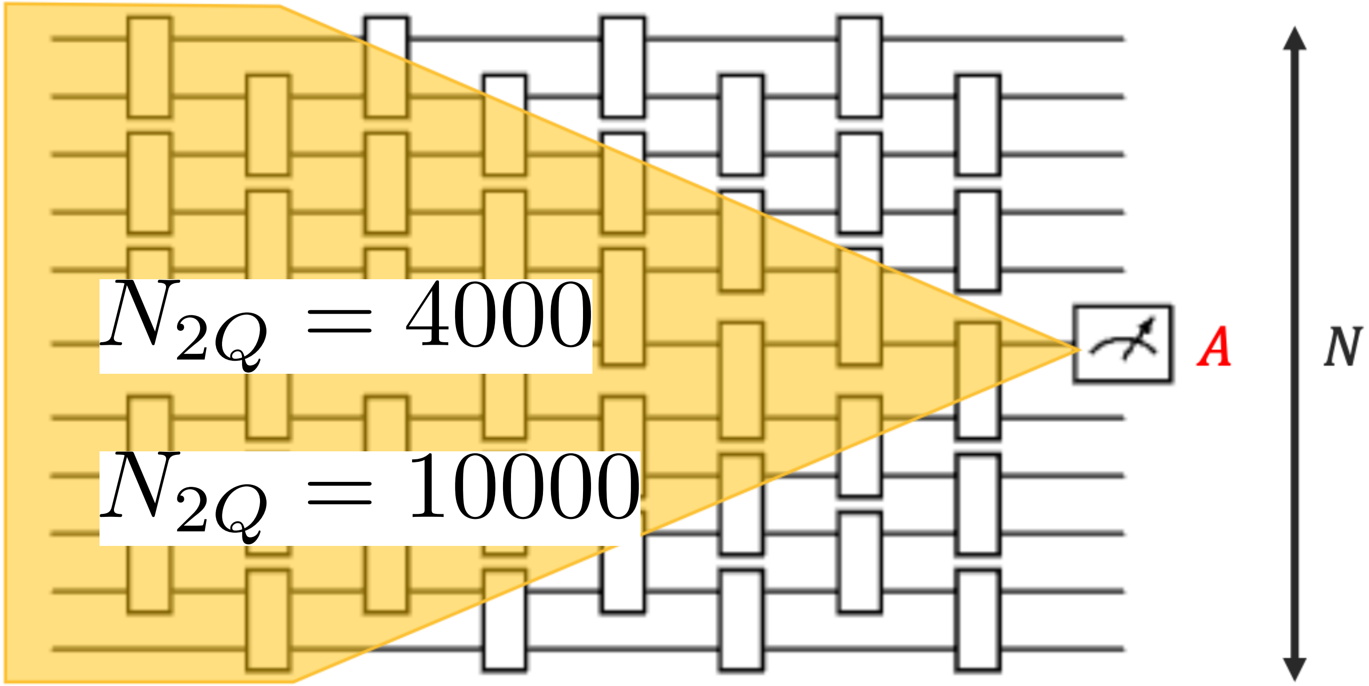
What can we do?

○ global depolarizing model $\langle \hat{O} \rangle_{\text{noisy}} = f \langle \hat{O} \rangle_{\text{ideal}}$

$$f = (1-p)^{N_{2Q}} = 0.135$$

$$p = 5 \times 10^{-4}$$

$$p = 2 \times 10^{-4}$$



$$N = 100, \quad dt = 0.1, \quad t = 10, \quad N_{2Q} \sim 10000$$

Hamiltonian simulation of LGTs is challenging in the NISQ era

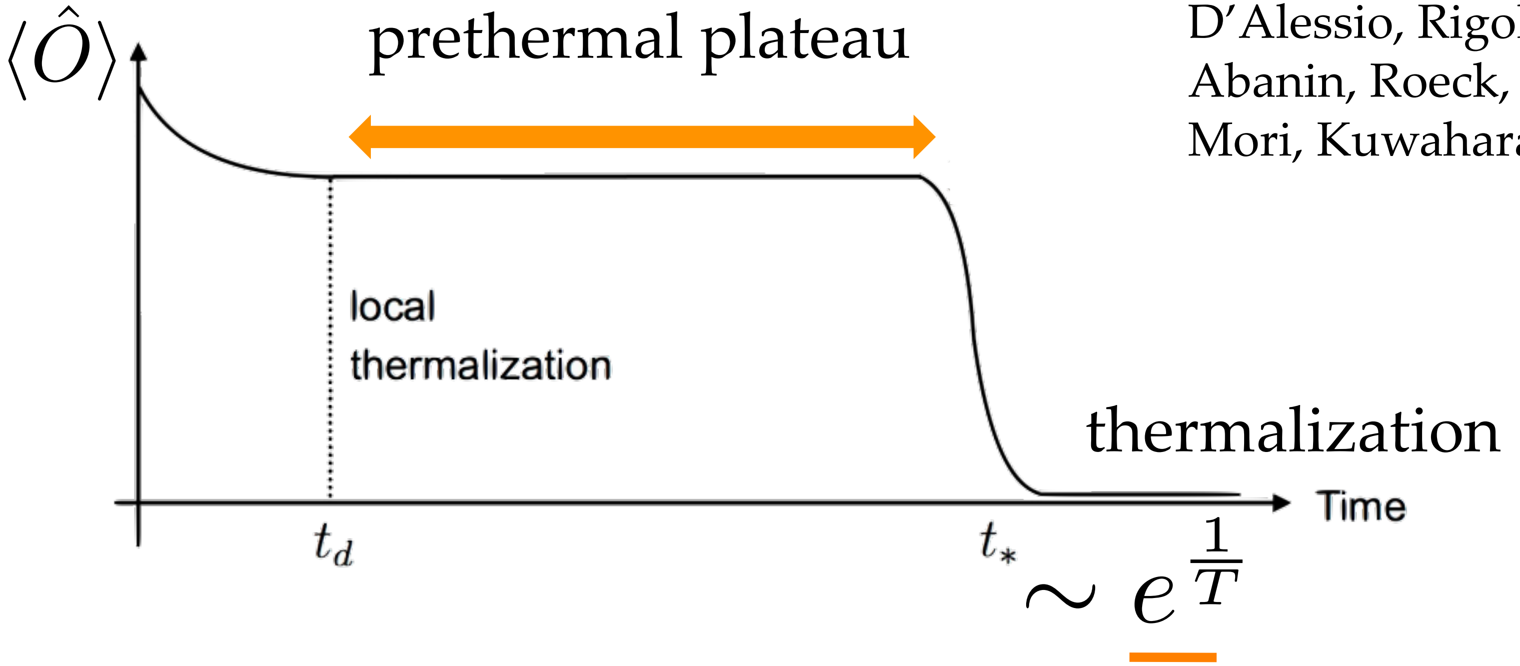
Floquet dynamics

○ Time evolution with a time-dependent Hamiltonian

$$i\partial_t|\Psi\rangle = H|\Psi\rangle \qquad H(t+T) = H(t)$$

driven system → heat up to infinite temperature

Lanzarides, Das, Moessner ’14
D’Alessio, Rigol ’14
Abanin, Roeck, Huveneers ’15
Mori, Kuwahara, Saito ’16



Trotter dynamics

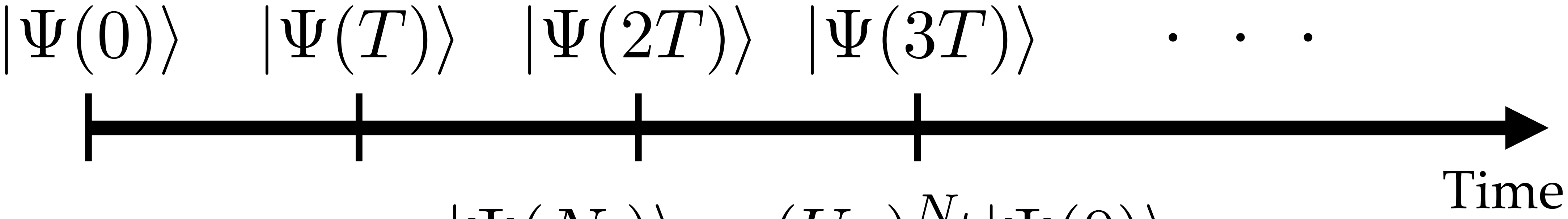
○ Trotter dynamics can be understood as Floquet dynamics

$$U_{\mathrm{F}} = \mathrm{e}^{-\mathrm{i}H_1 dt}\mathrm{e}^{-\mathrm{i}H_2 dt}$$

$$H(t) = \begin{cases} H_1 & t \in [0, T/2), \\ H_2 & t \in [T/2, T) \end{cases}$$

$$dt = \frac{T}{2}$$

$$i\partial_t|\Psi\rangle = H|\Psi\rangle \qquad H(t+T) = H(t)$$



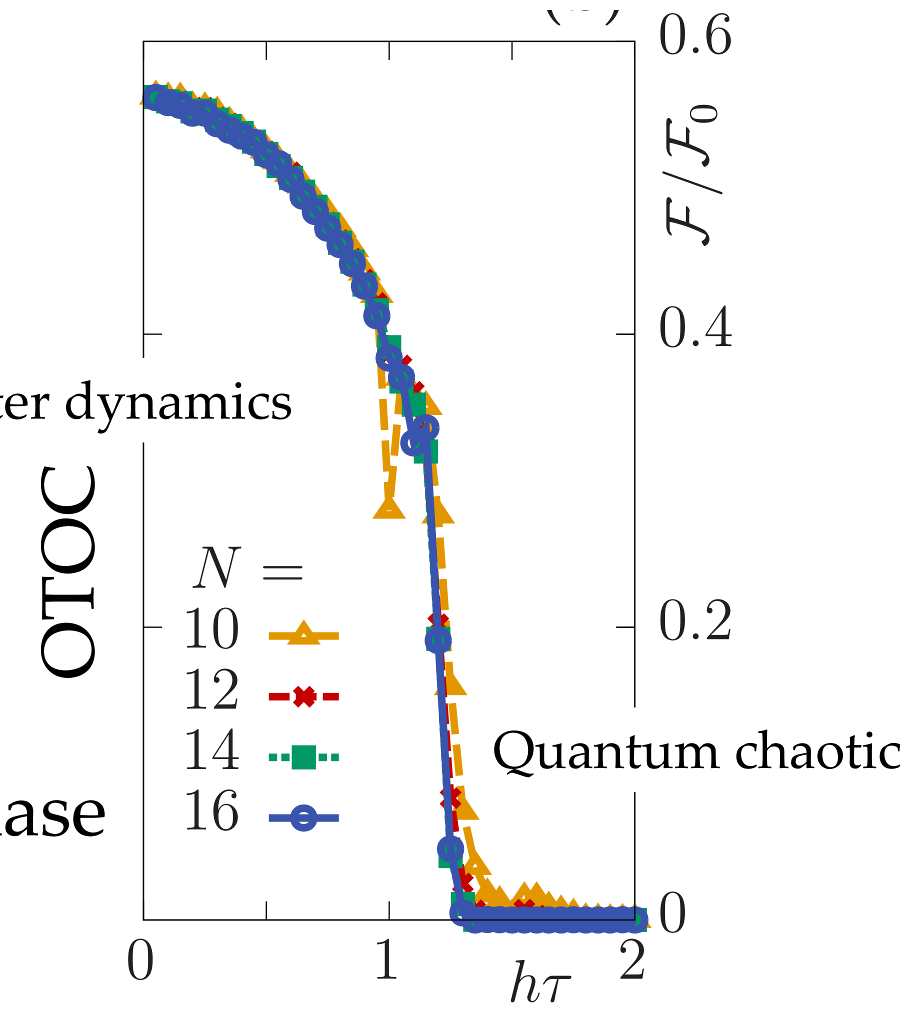
$$|\Psi(N_t)\rangle = (U_{\mathrm{F}})^{N_t}|\Psi(0)\rangle$$

Trotter dynamics

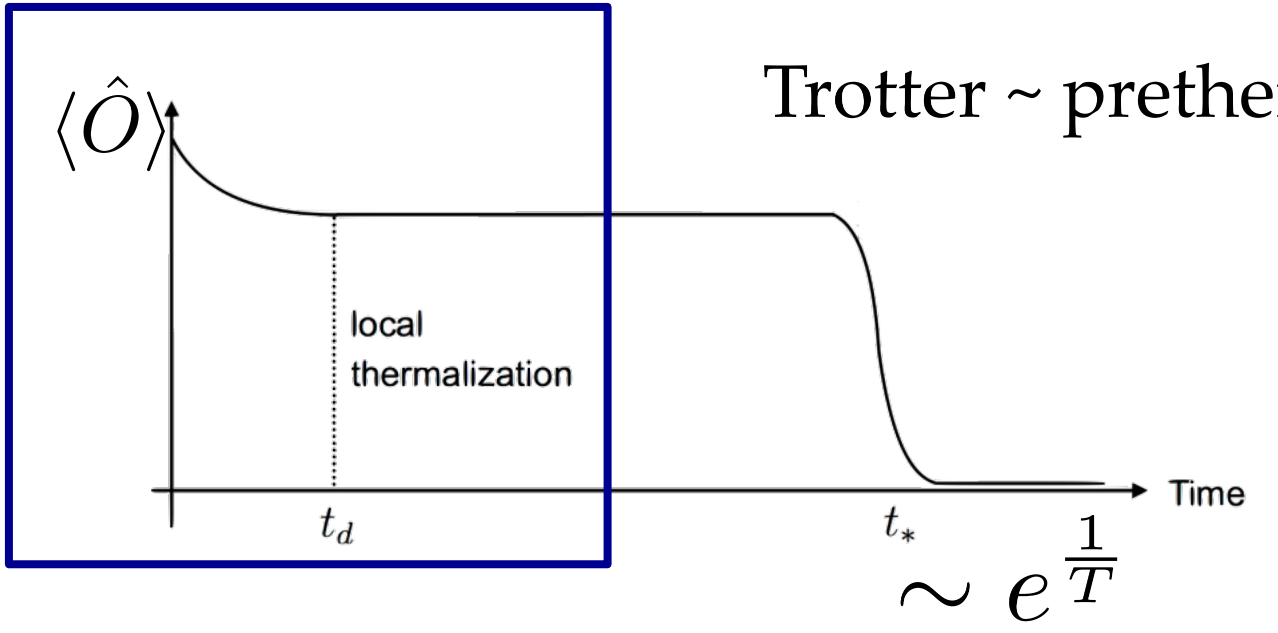
○ Trotter transition

Heyl, Hauke, Zoller '18
Varnier, Bertini, Giudici, Piroli '23

Trotter error is under control if $dt < dt_c$



Trotter ~ prethermal phase



prethermalizaion dynamics
is feasible to NISQ devices

Trotter dynamics as Floquet dynamics

○ Feasibility of information scrambling on the present best fidelity hardware

Ion traps: Kazuhiro Seki, Yuta Kikuchi, Tomoya Hayata, Seiji Yunoki, 2405.07613 [quant-ph]

○ Thermalization dynamics with many qubits

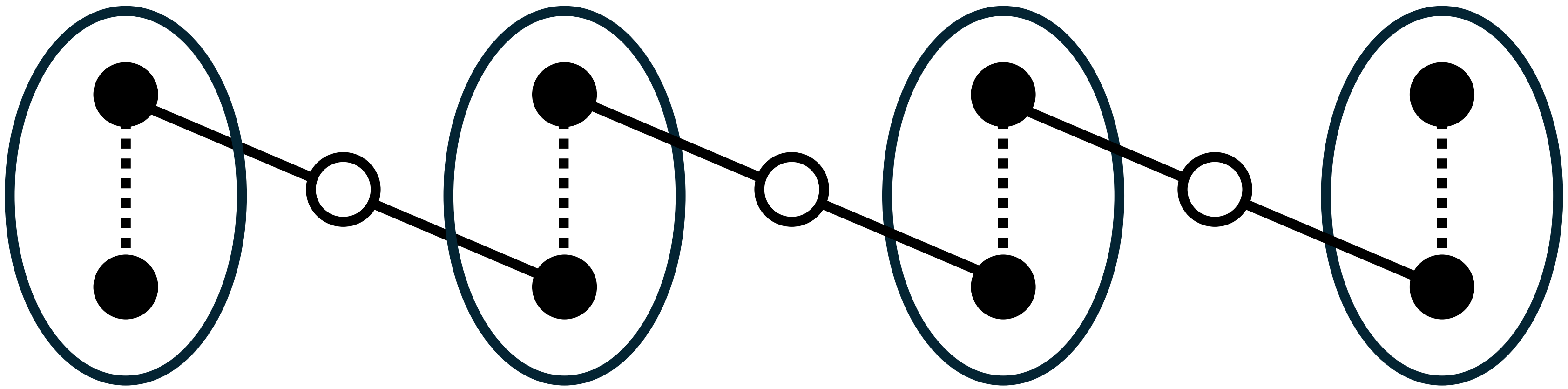
Superconducting qubits:
Tomoya Hayata, Kazuhiro Seki, Arata Yamamoto, arXiv:2408.10079 [hep-lat]
Tomoya Hayata, Yoshimasa Hidaka, 2409.20263 [hep-lat]

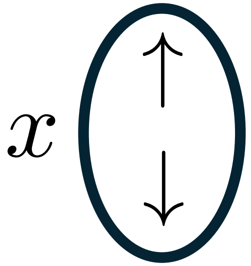
(1+1)-dimensional Z_2 LGT

Fermion

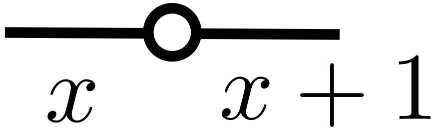
Z_2 Gauge field

Fermions with spins on one-dimensional lattice





Physical sites



Physical edges

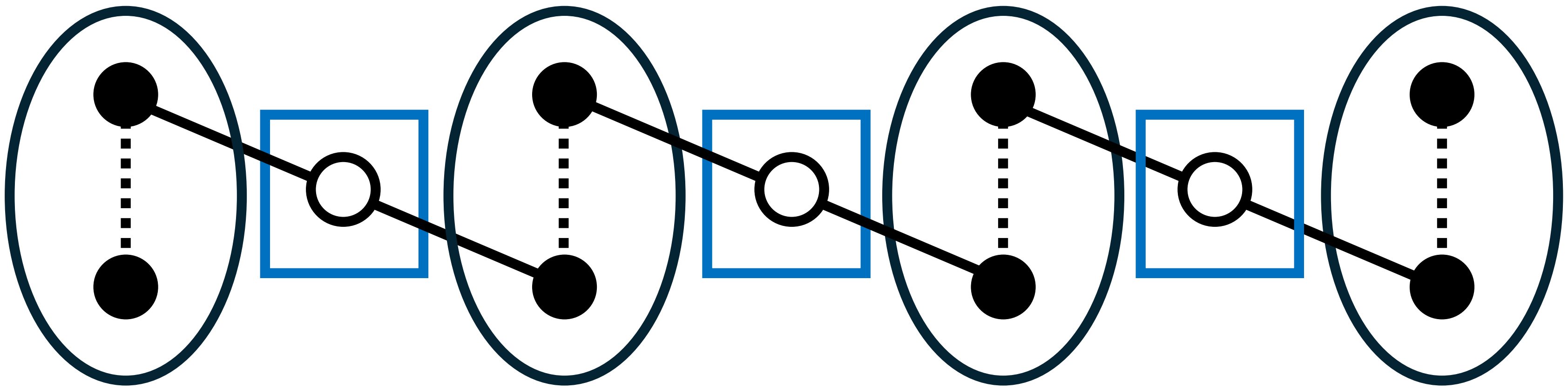
$$|\Psi\rangle = \prod_{x=1}^{N-1} |g(x, x+1)\rangle \prod_{x=1}^{N} |\psi_1(x)\rangle |\psi_2(x)\rangle$$

(1+1)-dimensional Z_2 LGT: Hamiltonian

Fermion

Z_2 Gauge field

Fermions with spins on one-dimensional lattice



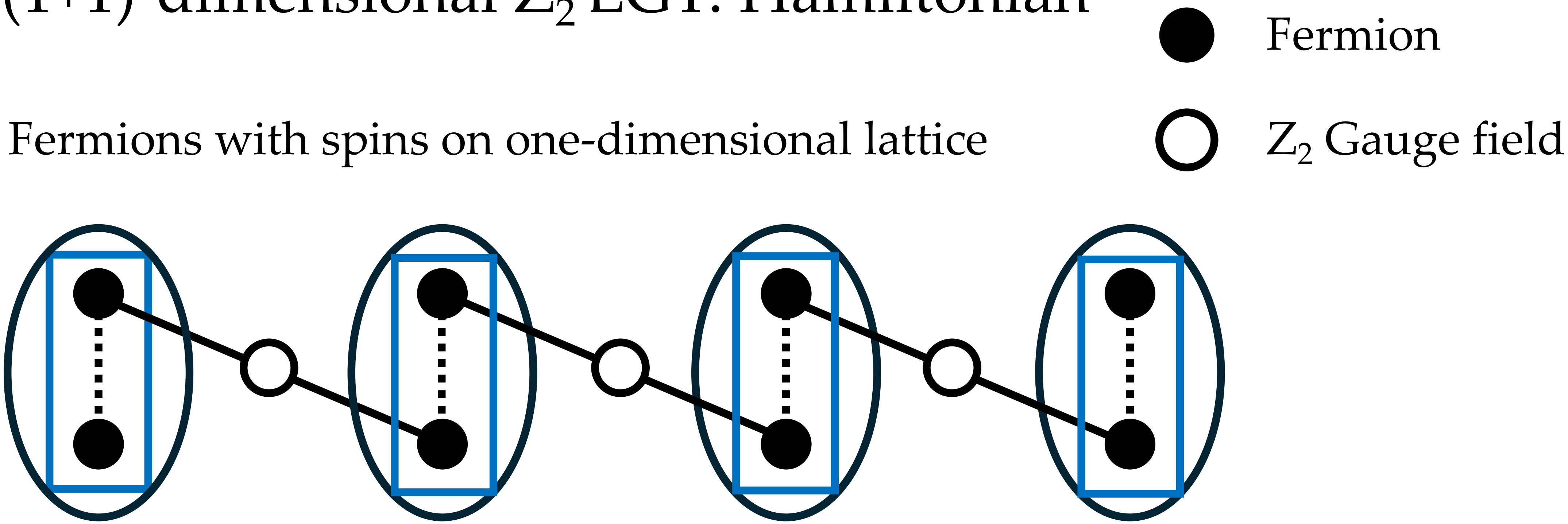
Electric field terms

$$H_g = -\underline{K} \sum_{x=1}^{N-1} X_g(x, x+1)$$

string tension

(1+1)-dimensional Z_2 LGT: Hamiltonian

Fermions with spins on one-dimensional lattice



Fermion mass terms

$$H_f = \sum_{x=1}^{N} (1 + \underline{m}) \psi^\dagger(x) \gamma^0 \psi(x)$$

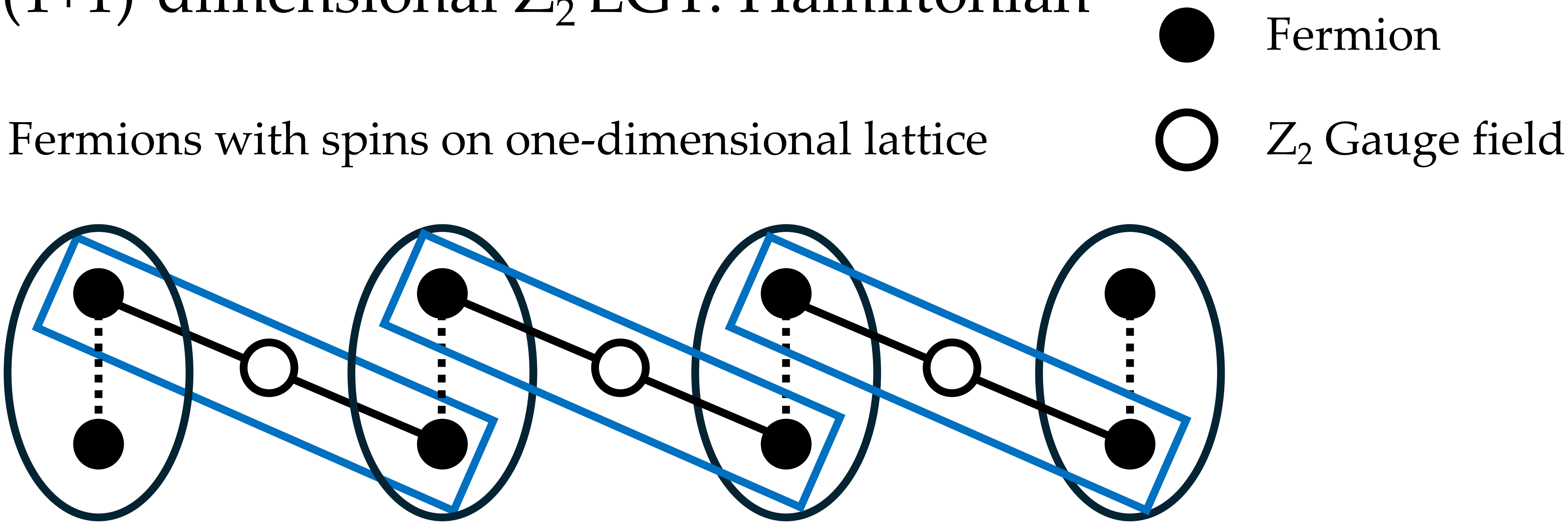
mass

$$\gamma_0 = \sigma_x$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

(1+1)-dimensional Z_2 LGT: Hamiltonian

Fermions with spins on one-dimensional lattice

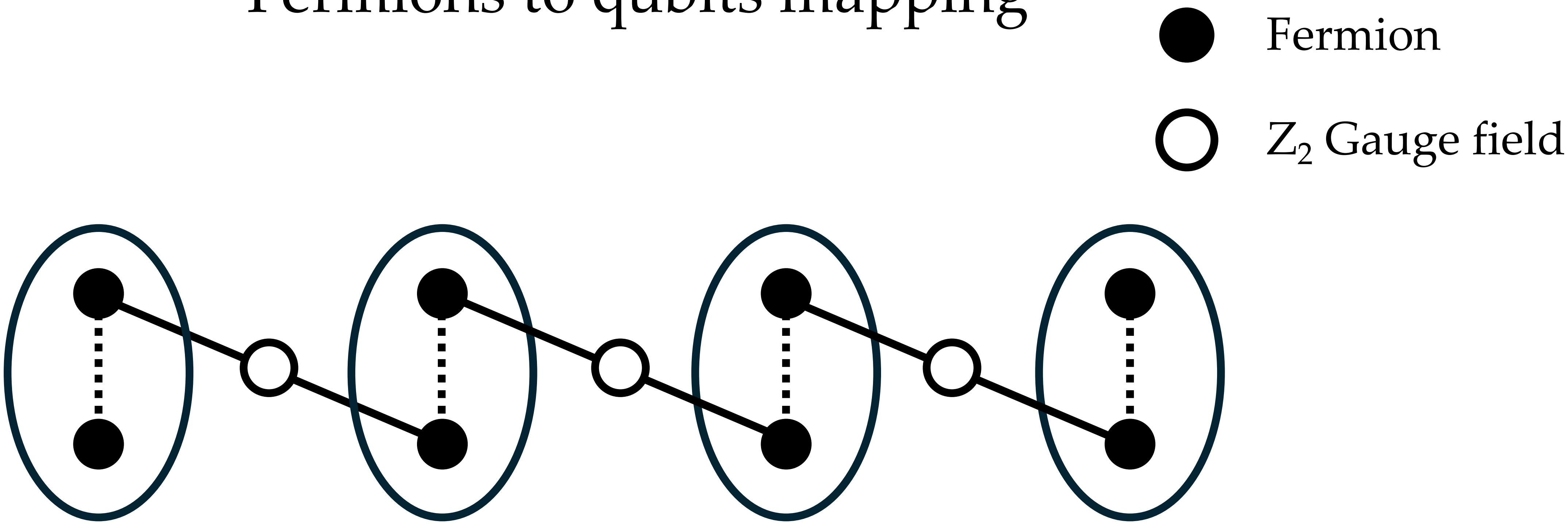


Fermion hopping terms

$$\gamma_0 = \sigma_x$$
$$\gamma_1 = \sigma_z$$

$$H_{gf} =$$
$$-\frac{1}{2}\sum_{x=1}^{N-1} Z_g(x,x+1)\{\psi^\dagger(x)\gamma^0(1-\gamma^1)\psi(x+1) + \psi^\dagger(x+1)\gamma^0(1+\gamma^1)\psi(x)\}$$

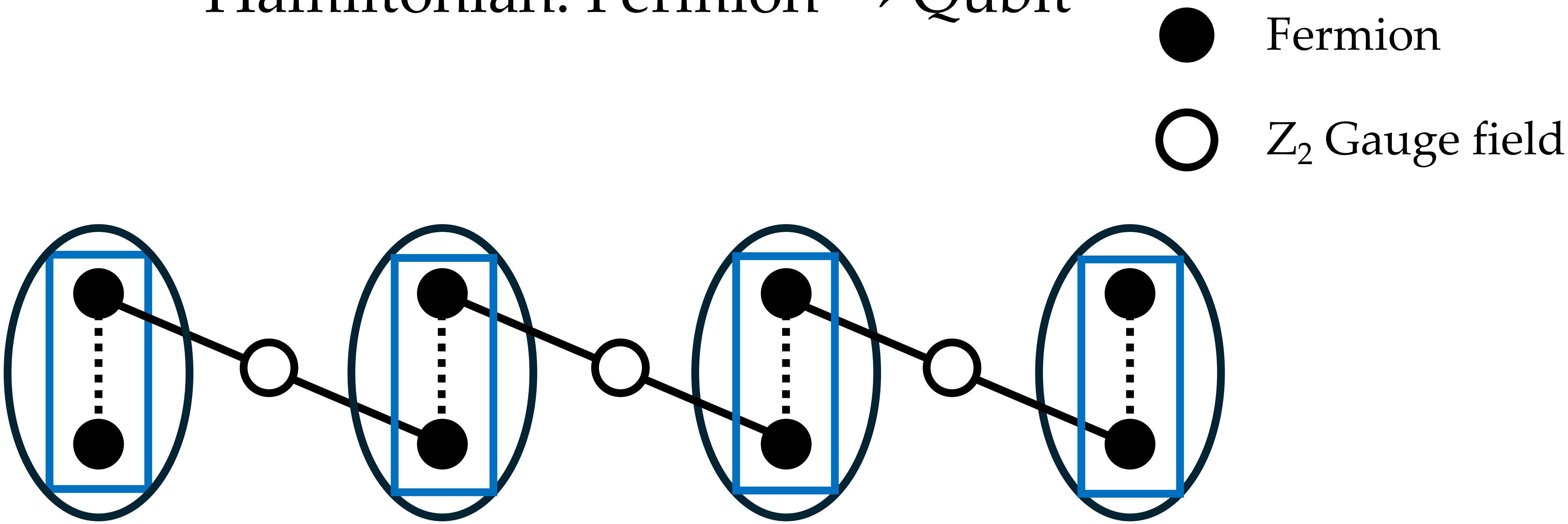
Fermions to qubits mapping



Only nearest neighbor hoppings of 1d spinless fermions

→ Jordan-Wigner transformation works perfectly

Hamiltonian: Fermion → Qubit

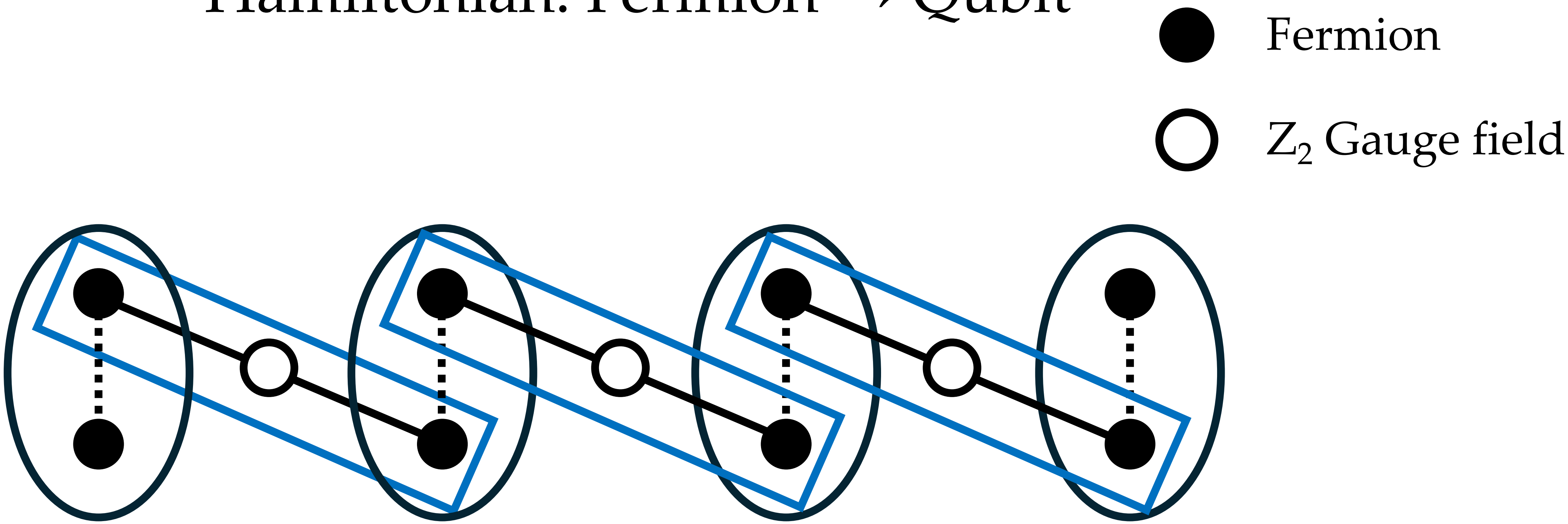


Fermion mass terms

$$H_f = \frac{1}{2}\sum_{x=1}^{N}(1+m)\left\{X_1(x)X_2(x)+Y_1(x)Y_2(x)\right\}$$

Spinless Fermion NN hopping terms

Hamiltonian: Fermion → Qubit

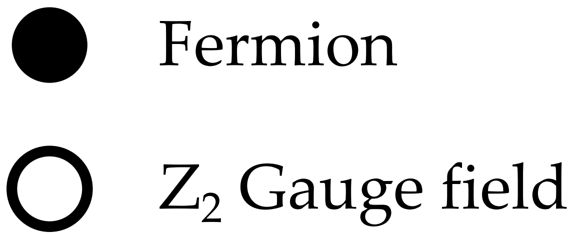


Fermion hopping terms

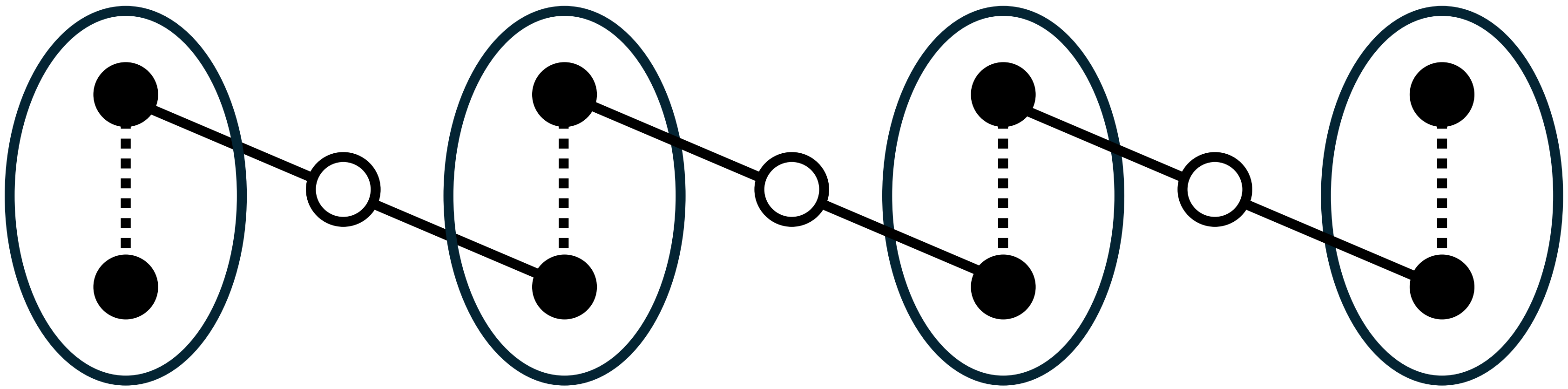
$$H_{gf} = -\frac{1}{2}\sum_{x=1}^{N-1} Z_g(x,x+1)\left\{X_1(x)X_2(x+1) + Y_1(x)Y_2(x+1)\right\}$$

Spinless Fermion NN hopping terms

Hamiltonian: summary



N-site Z_2 LGT → 3*N*-site qubit chain



Hamiltonian: summary

● Fermion

○ Z_2 Gauge field

N-site Z_2 LGT → $3N$-site qubit chain

$$H = H_g + H_f + H_{gf}$$

$$H_g = -K \sum_{x=1}^{N-1} X_g(x, x+1)$$

$$H_f = \frac{1}{2} \sum_{x=1}^{N} (1+m) \{X_1(x)X_2(x) + Y_1(x)Y_2(x)\}$$

$$H_{gf} = -\frac{1}{2} \sum_{x=1}^{N-1} Z_g(x, x+1) \{X_1(x)X_2(x+1) + Y_1(x)Y_2(x+1)\}$$

Floquet circuit from Suzuki-Trotter decompositon

1^{st} order Suzuki-Trotter decomposition

makes *dt* very large

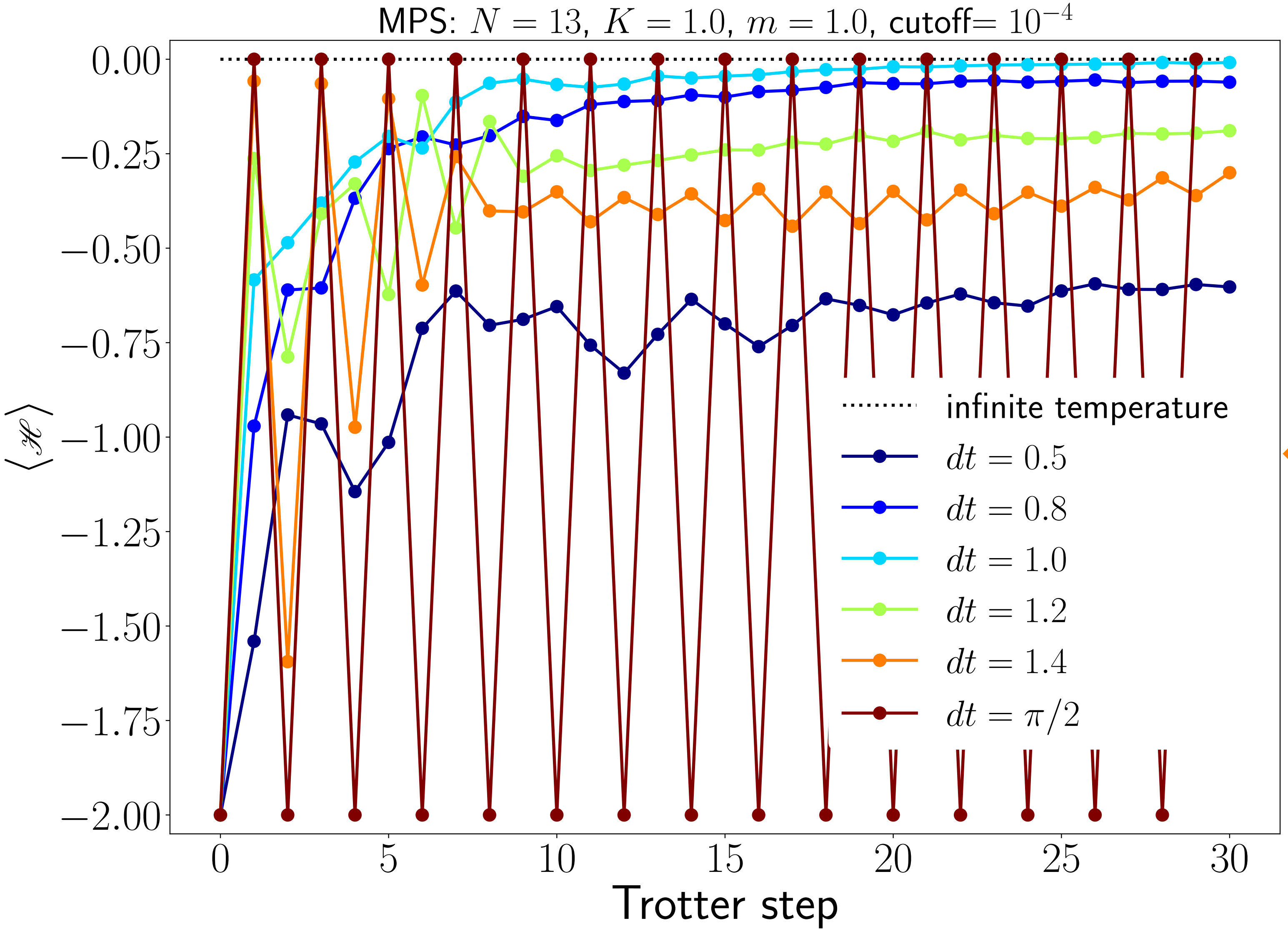
$$U_{\mathrm{F}} = \mathrm{e}^{-\mathrm{i}H_{gf}dt}\mathrm{e}^{-\mathrm{i}(H_f+H_g)dt}$$

Floquet evolution

$$|\Psi(N_t)\rangle = (U_{\mathrm{F}})^{N_t}|\Psi(0)\rangle$$

Initial state is the groundstate of H_f+H_g

MPS results



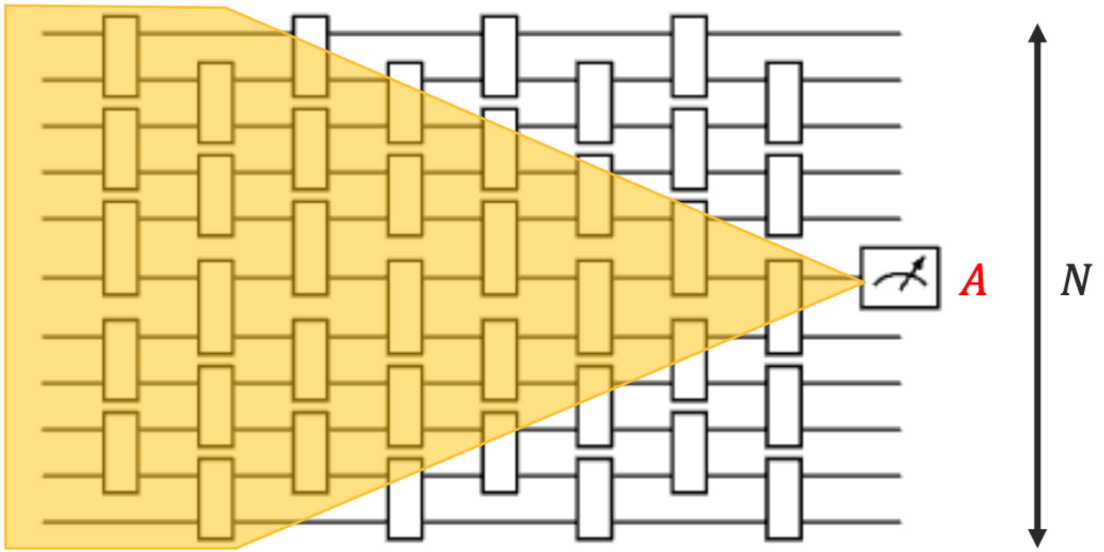
H_f Fermion mass term at the center of chain

2qubit operator

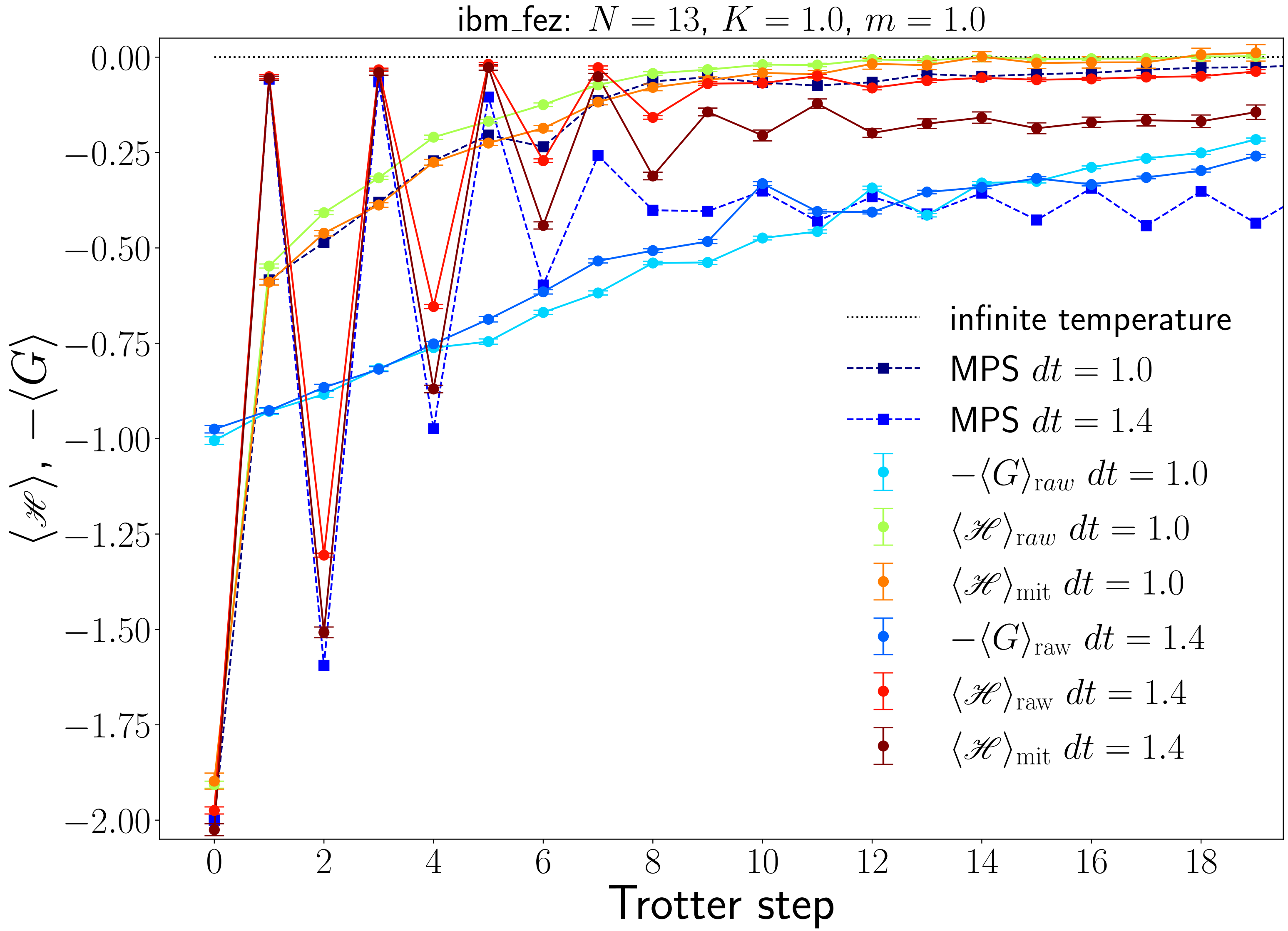
Hamiltonian

QC summary

- $dt = 1.0,\ 1.4$
- ibm_fez is used
- 10,000 shot is used
- Pauli twirling is enabled/ DD is disabled
- Unnecessary gates outside of the causal cone are removed



Experimental results (raw data)



H_f Fermion mass term at the center of chain

2qubit operator

G Gauss's law operator at the center of chain

4qubit operator

Successful only the very short time and error mitigation is inevitable

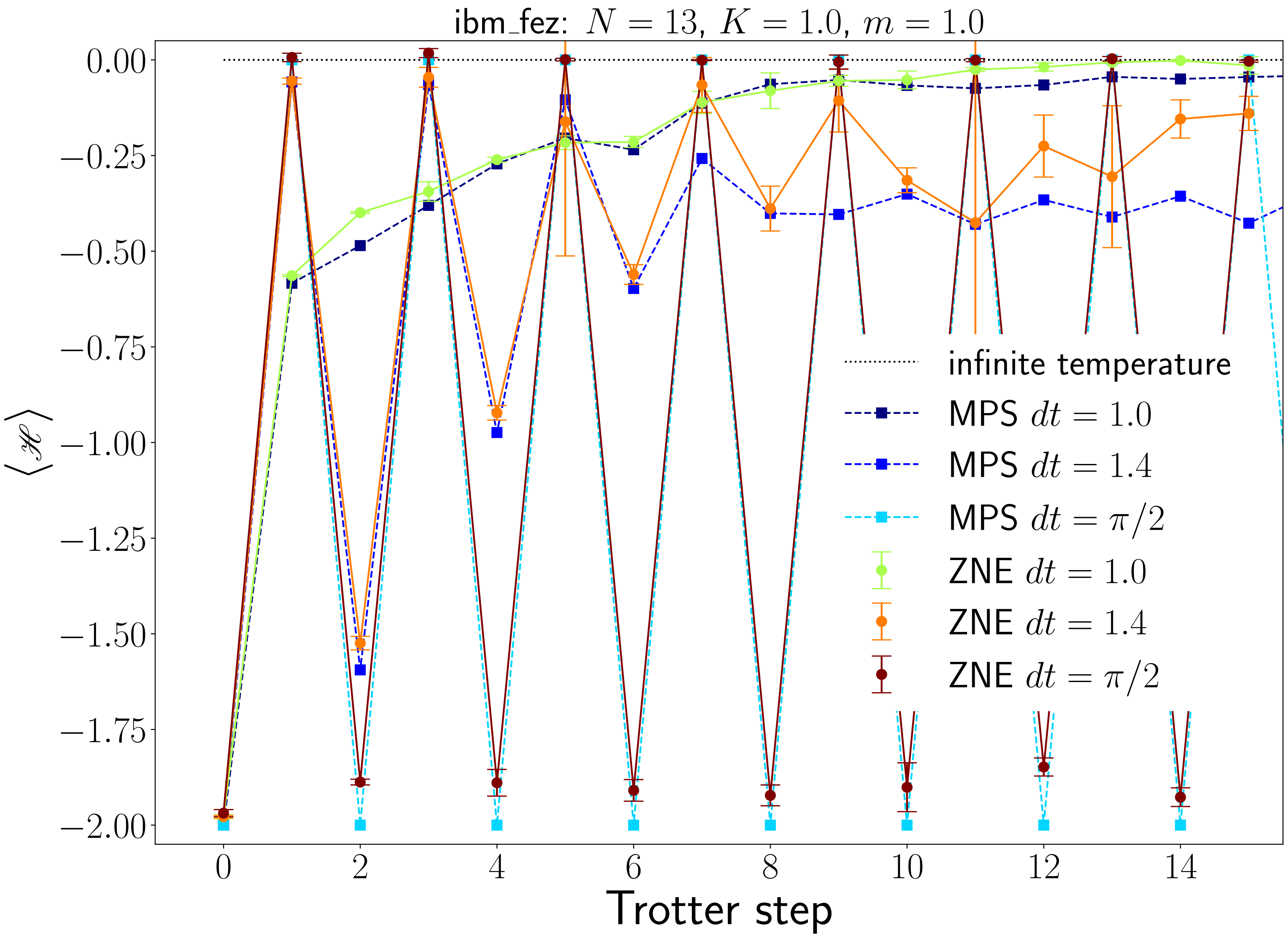
Quantum error mitigation (ZNE)

○ running twirled circuits $U(U^\dagger U)^{\frac{\mathcal{N}-1}{2}}$ $\mathcal{N} = 1, 3, 5$

○ extrapolate to the zero noise limit

○ Qistkit's native ZNE function is used

Quantum error mitigation (ZNE)



Successful in running Floquet circuit with $N_t = 10$

(Early stage of)
Thermal plateaus are observed

Pauli twirling is important

ZNE is the most stable

IBM machine has the capability of simulating the physical phenomena

Quantum error mitigation (Gauss's laws)

- assume the global depolarizing channel as a noise model

$$\langle \hat{O} \rangle_{\mathrm{raw}} = f \langle \hat{O} \rangle_{\mathrm{ideal}} + \frac{1-f}{2^{3N-1}} \mathrm{Tr}\hat{O}$$

- estimate f by measuring Gauss's law

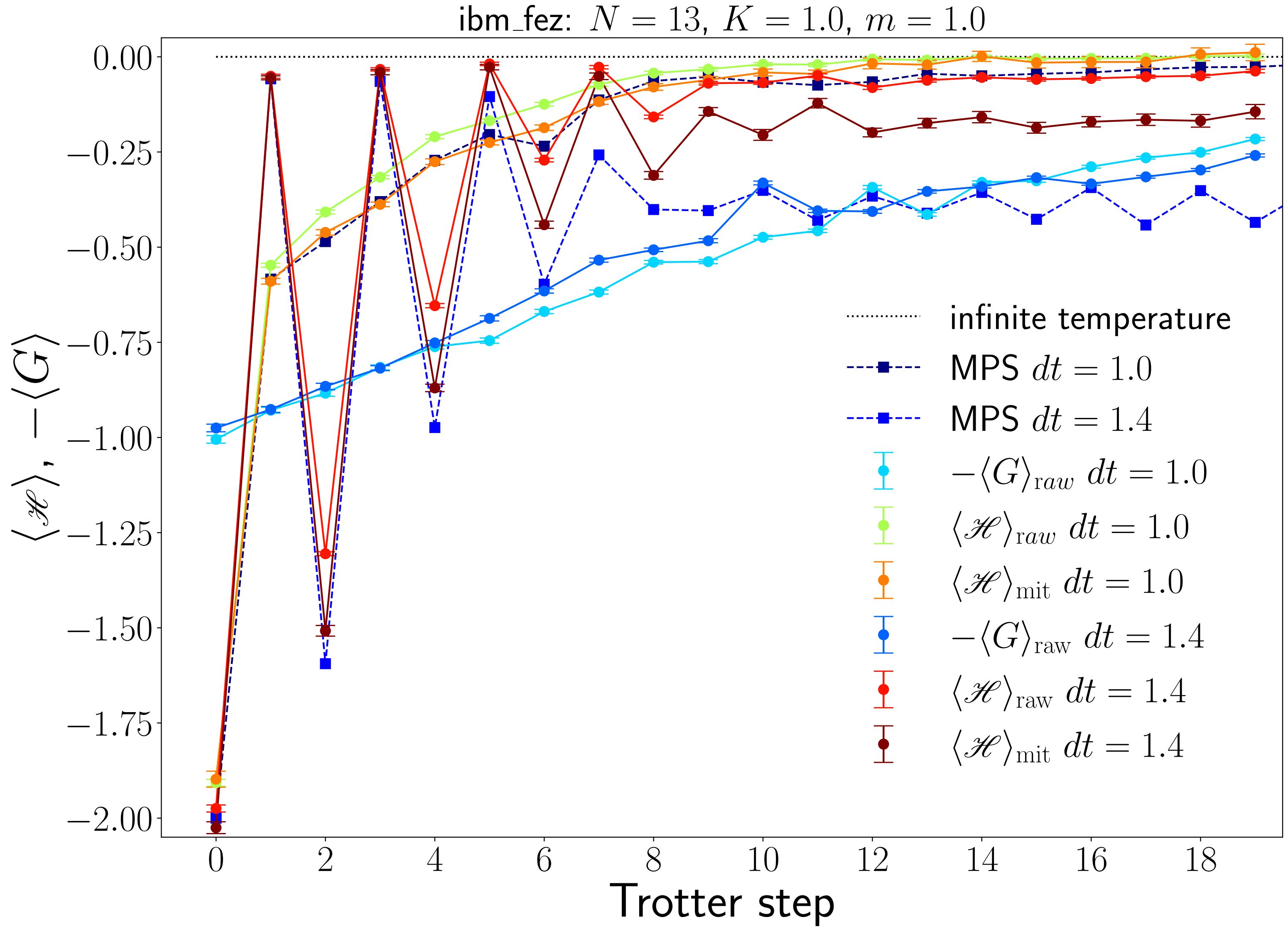
$$\langle \hat{G} \rangle_{\mathrm{ideal}} = 1 \quad \longrightarrow \quad \langle \hat{G} \rangle_{\mathrm{noisy}} = f$$

- rescale observables by

$$\langle \mathscr{H}(x_c) \rangle_{\mathrm{mit}} = \frac{\langle \mathscr{H}(x_c) \rangle_{\mathrm{raw}}}{\langle G(x_c) \rangle_{\mathrm{raw}}}$$

- is unique to LGT and computationally very cheap

Quantum error mitigation (Gauss's laws)



Successful in running Floquet circuit with $N_t = 10$

Pauli twirling is important

This is less good but still works/ Computational cost is much cheaper

Quantum error mitigation (ODR)

- assume the global depolarizing channel as a noise model

$$\langle\hat{O}\rangle_{\rm raw} = f\langle\hat{O}\rangle_{\rm ideal} + \frac{1-f}{2^{3N-1}}{\rm Tr}\hat{O}$$

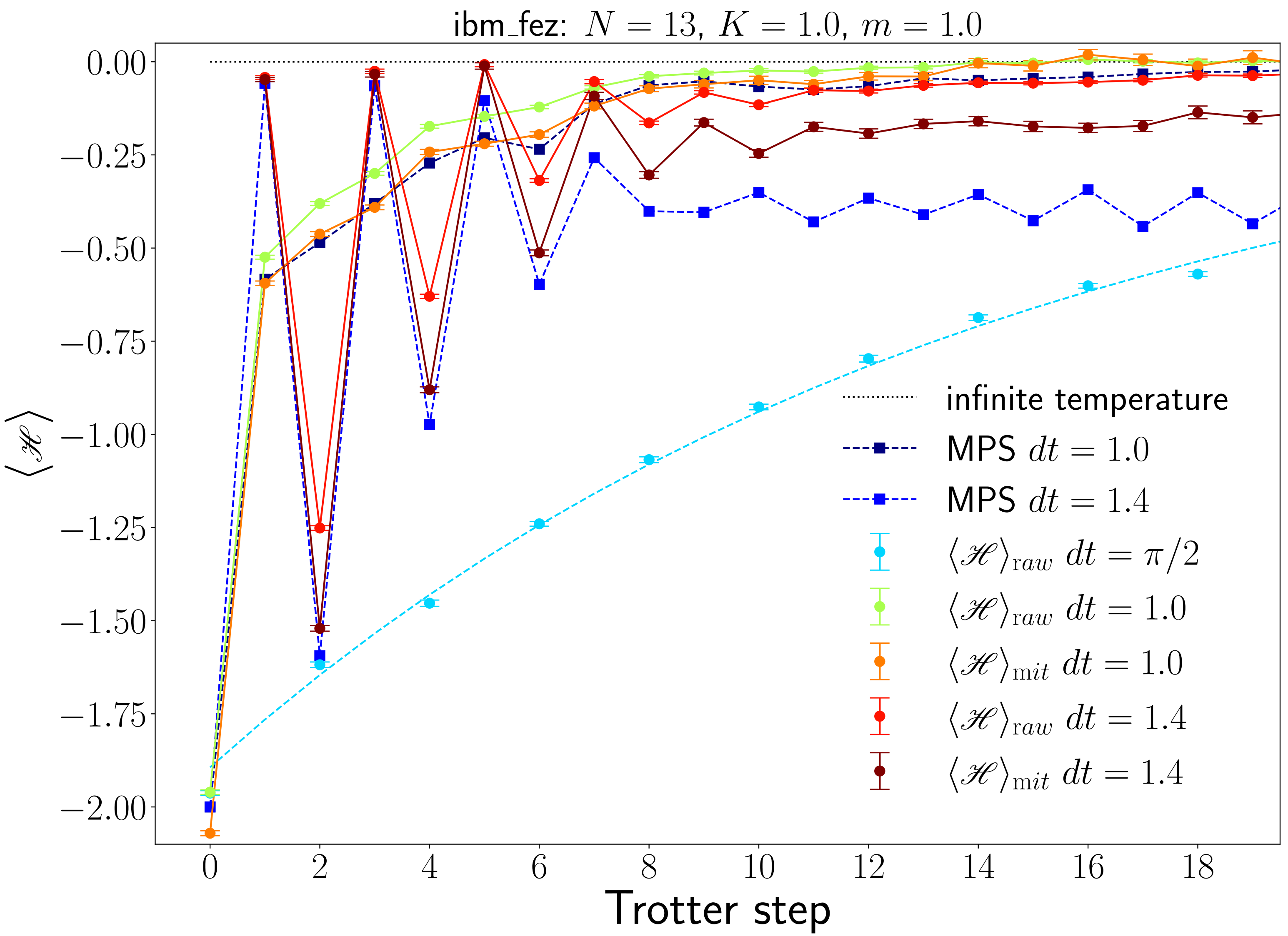
- run the same circuit with the “solvable” parameter

- rescale observables by

$$\langle\mathscr{H}(x_c)\rangle_{\rm mit} = \frac{\langle\mathscr{H}(x_c)\rangle_{\rm raw}}{f}$$

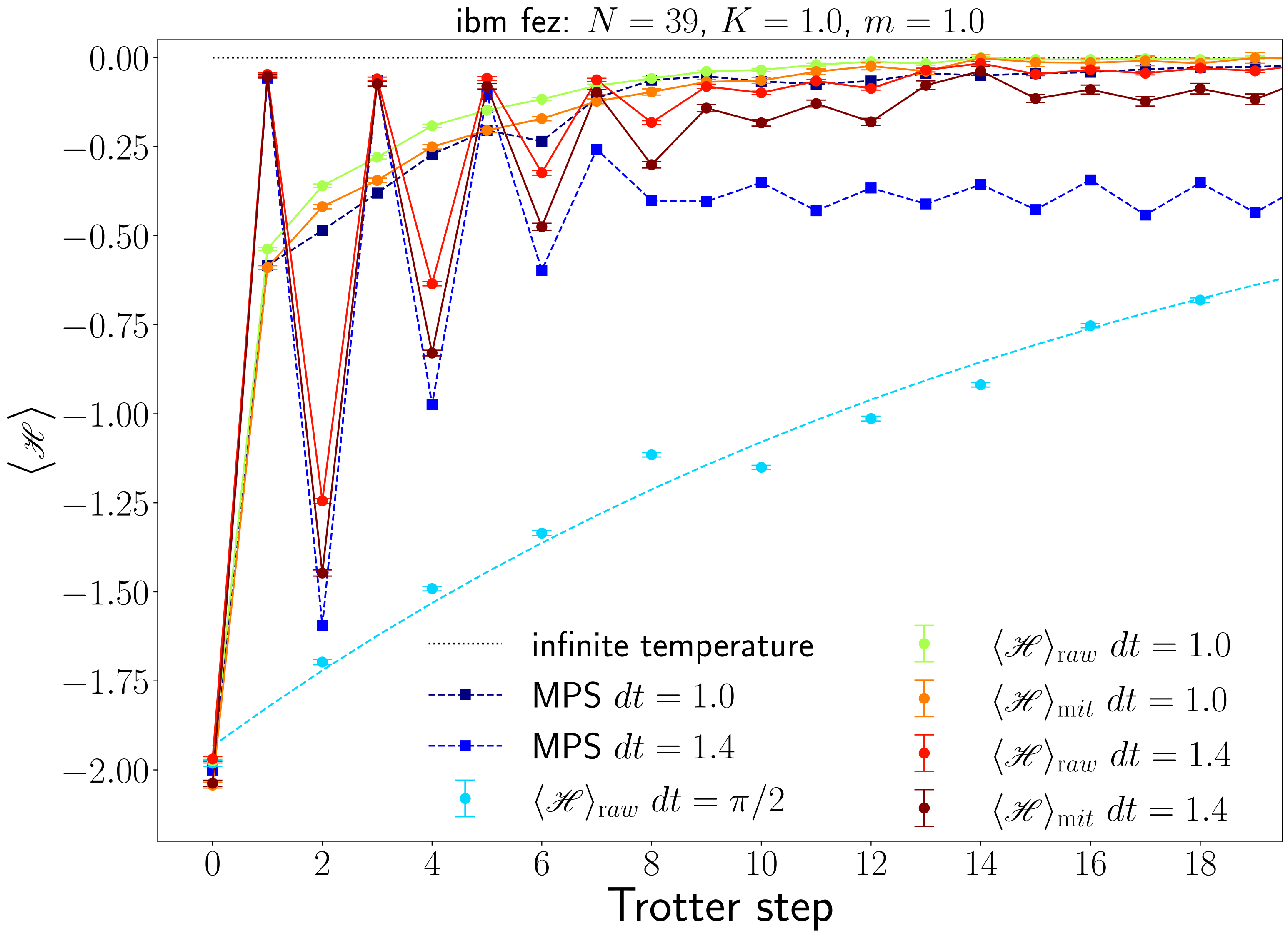
- is computationally very cheap

Quantum error mitigation (ODR)



This is less good but still works/ Computational cost is much cheaper

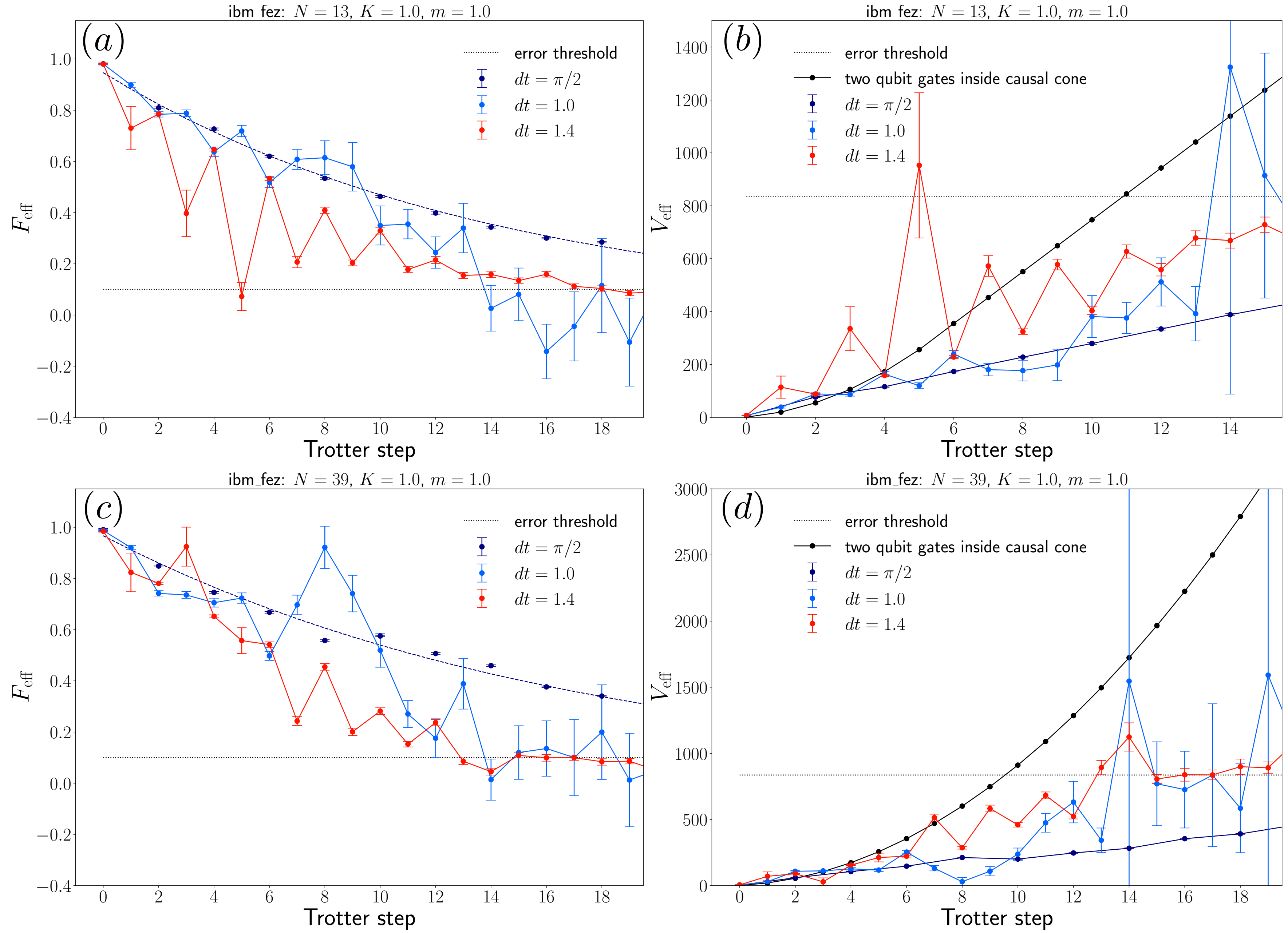
Quantum error mitigation (ODR): N>100 qubits



Successful in running Floquet circuit with $N_t = 10$ and beyond 100 qubits

We can simulate the physical phenomena in the quantum utility scale

Fidelity and circuit volume



$$F_{\mathrm{eff}} = (1-p)^{V_{\mathrm{eff}}}$$

$$p = 2.75 \times 10^{-3}$$

Growth of V_{eff} is very slow

Thermalization dynamics may be useful for showing quantum advantage

Summary

○ QC for QFT is anticipated but challenging

○ Quantum simulation of Floquet circuits in near future devices is interesting and may be useful for showing quantum advantage

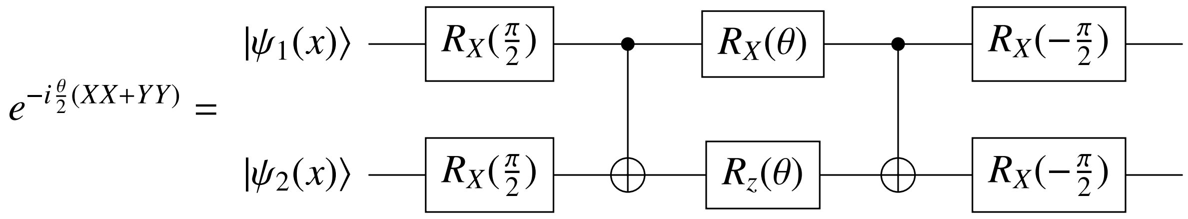
○ Lattice gauge theories have complex Hamiltonians and may provide good playgrounds for testing the capability of QCs

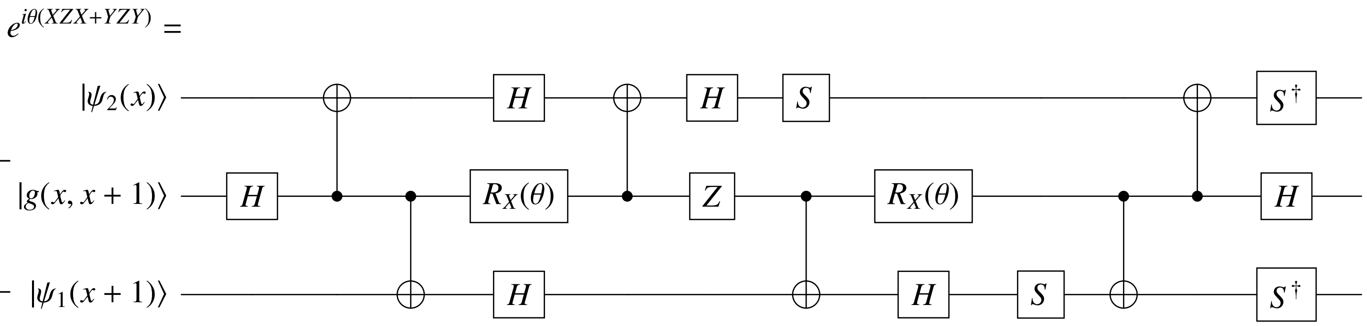
Floquet circuit from Suzuki-Trotter decompositon

1[st] order Suzuki-Trotter decomposition

makes dt very large

$$U_{\mathrm{F}} = \mathrm{e}^{-\mathrm{i}H_{gf}dt}\mathrm{e}^{-\mathrm{i}(H_f+H_g)dt}$$





In total, $(8N-6)N_t + N$ CZ gates

Initial state is the groundstate of H_f+H_g