State preparation and operator growth of SYK model on IBM quantum computer

arXiv: 2311.17991 (Phys.Rev. D 109, 105002) [with Asad, B. Sambasivam]
arXiv: 2406.15545 [in review, with Jack Araz, F. Ringer, B. Sambasivam]

Tensor Network 2024
November 17

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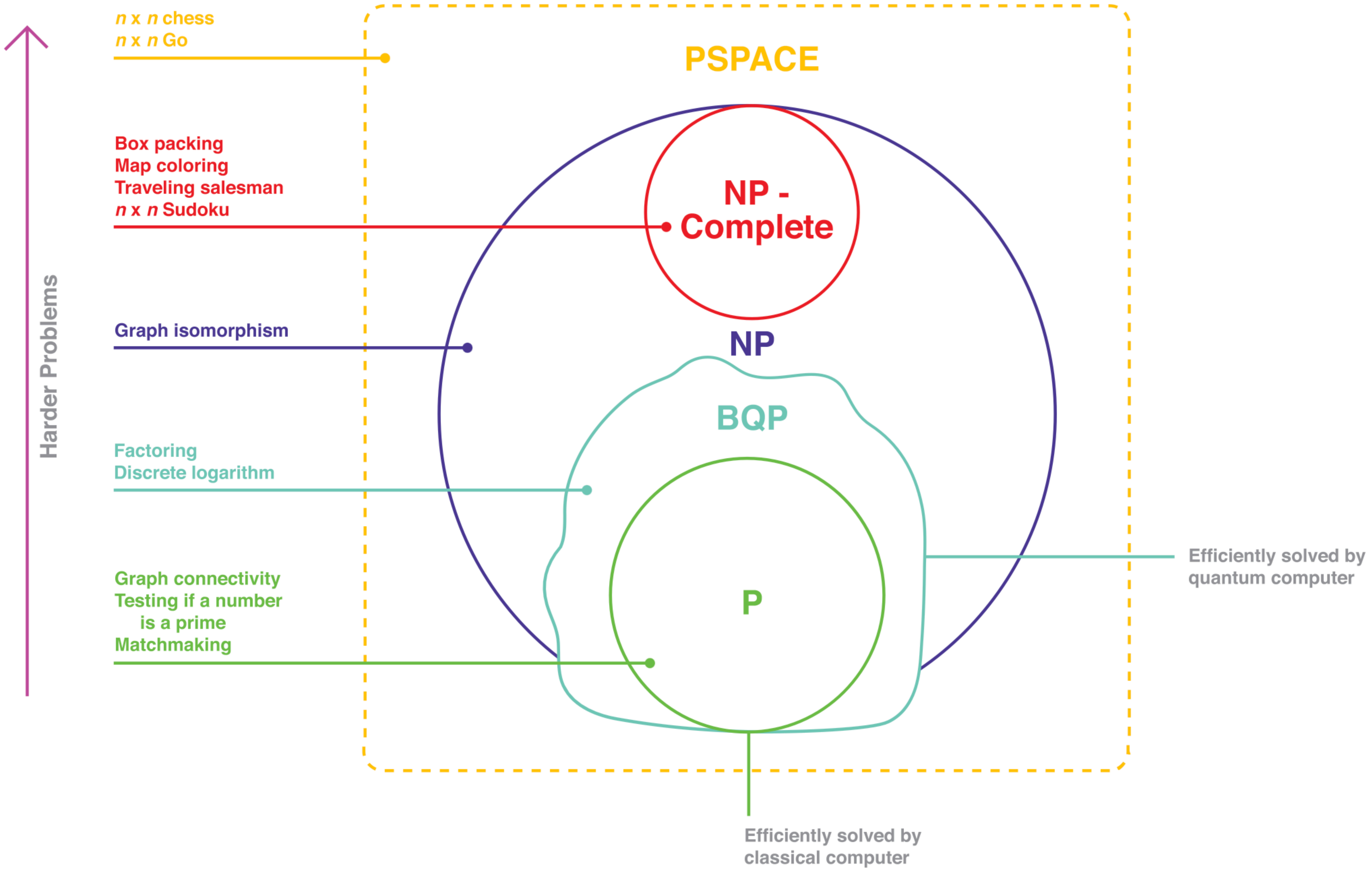
rgjha.github.io

Complexity


n x n chess
n x n Go
PSPACE
Box packing
Map coloring
Traveling salesman
n x n Sudoku
NP -
Complete
Graph isomorphism
NP
Harder Problems
BQP
Factoring
Discrete logarithm
Graph connectivity
Testing if a number
is a prime
Matchmaking
P
Efficiently solved by
quantum computer
Efficiently solved by
classical computer

Misconception: QC **can** solve all problems

- It turns out that for majority of problems, quantum computers would do no better than classical computers. A major research direction is to understand which problems can be solved efficiently by QCs.

- For example, we know that scattering in ϕ^4 can be solved efficiently by quantum computers arXiv: 1703.00454

- Class of problems which are best suited for quantum advantage belong to complexity class BQP. For ex: Shor's algorithm. Also Grover's algorithm but not as nice as Shor's (only polynomial speed-up).

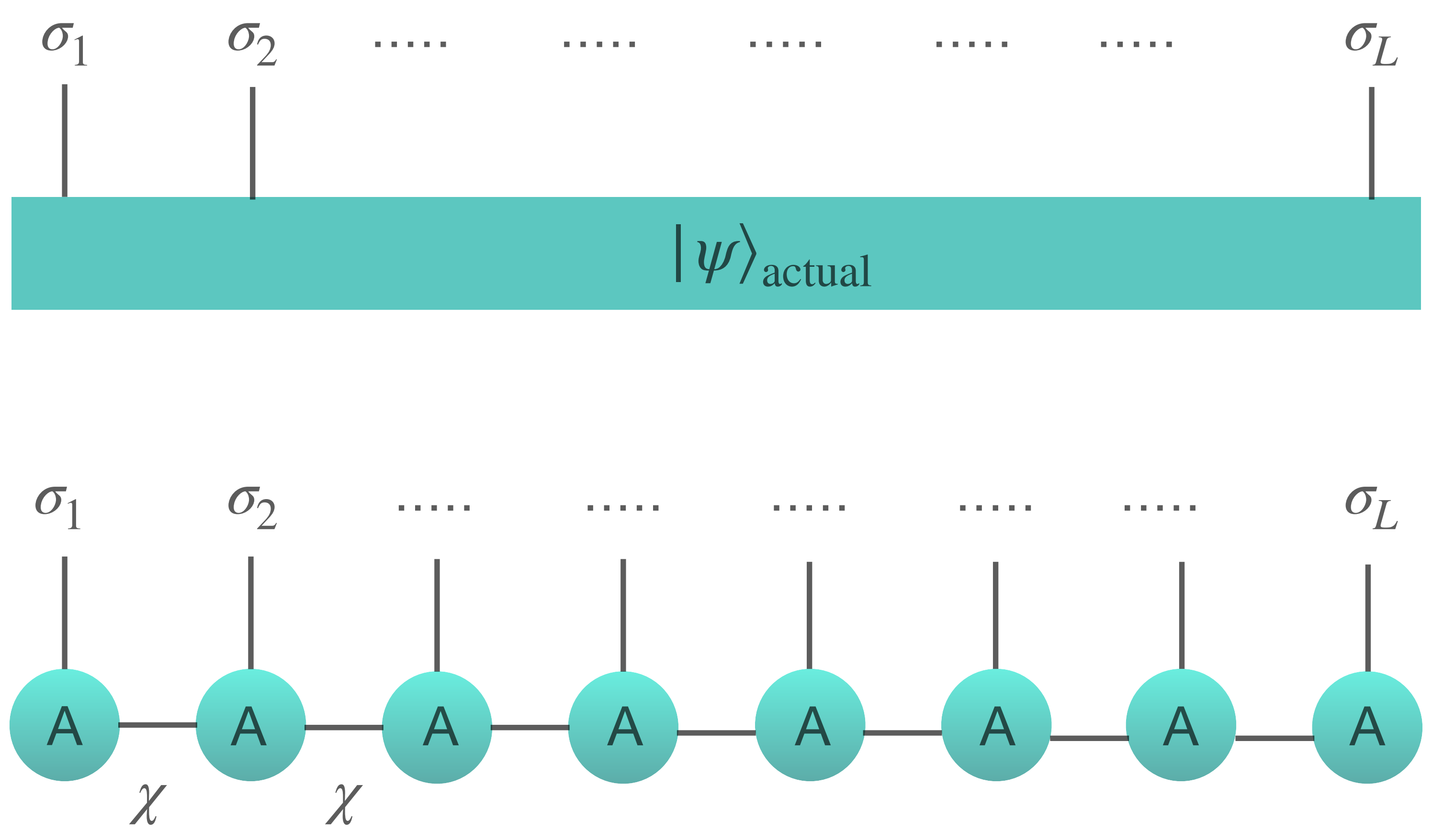
Outline of the talk

- Effectiveness of tensor networks - MPS approximation and towards universal *quantum*
- Sachdev-Ye-Kitaev (SYK) model of holography
- Quantum gates and real-time evolution using quantum circuits
- SYK model with N = 6, 8 Majorana fermions on IBM quantum computers with error mitigation - real-time dynamics and thermal state preparation
- Summary and future directions

Tensor networks

- The most efficient classical method of studying the properties of lower-dimensional systems is tensor networks. The idea is based on the fact that if the Hamiltonian is sufficiently local and gapped, then the relevant sector of the entire Hilbert space is a tiny region which satisfies area-law entanglement i.e., they are less entangled.

- In this case, the vector space of dimension d^N can be described by $O(d\chi^2)$ where χ is the bond dimension of the MPS. This prescription fails for gapless systems and has to be replaced by more complicated network such as MERA.

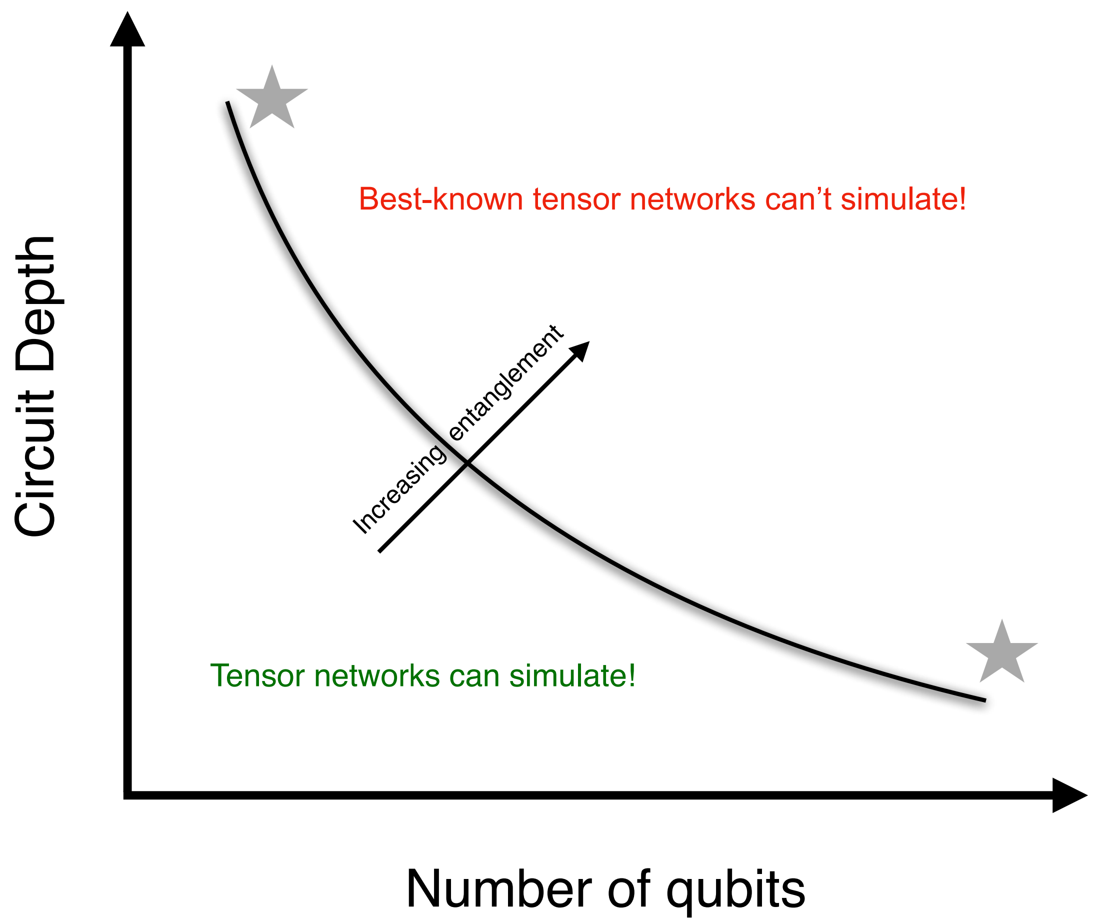


Classical to Quantum

- An important ingredient of numerical lattice formalism is Wick rotation. Can't use sampling methods otherwise.

- Tensor networks can help sometimes but they have their own limitations. We need new tools to understand real-time dynamics of interacting field theories or quantum many-body systems.

- We require fundamentally *new* idea of computing [Manin, Benioff, Feynman et al., circa 1978] such that we can compute exp(-iHt) for a given H in terms of circuits exploiting features of QM more efficiently than classical computers.

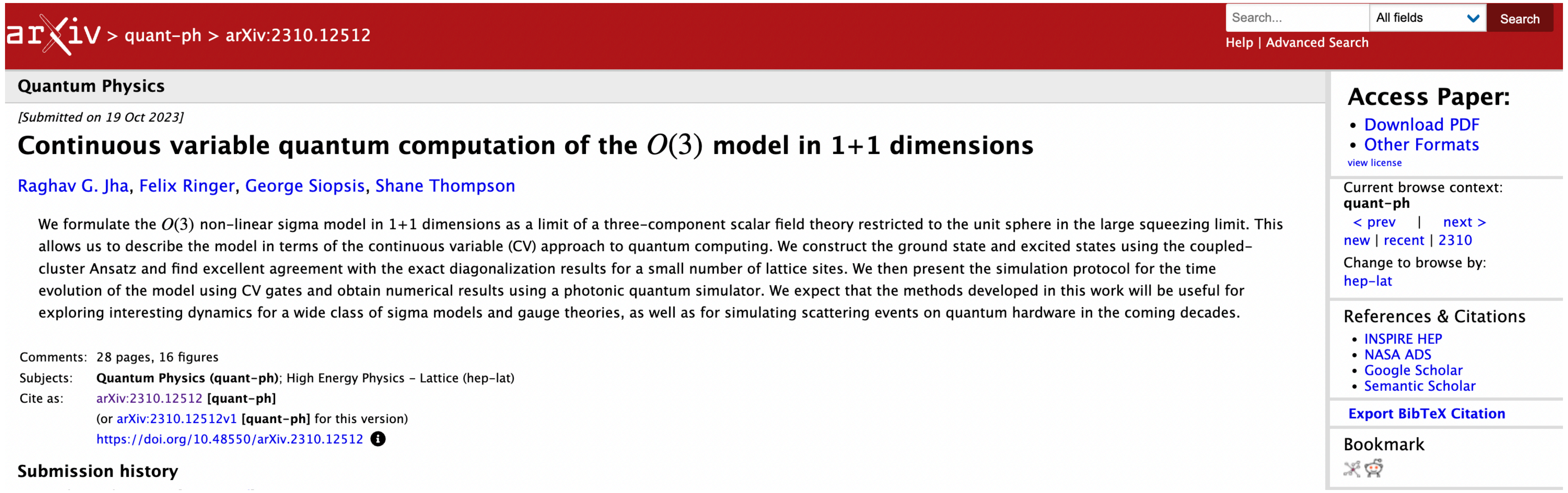


Approaches to universal quantum computing

- Qubit approach — Manipulate and utilize the two-state quantum system. More than dozen approaches. Two most popular — Superconducting and Trapped Ion.

- Qumodes approach — Use photons (quantum harmonic oscillator), infinite-dimensional HS. Not as popular as qubit approach. Error correction not that well-developed.

- This talk will discuss the qubit approach, however, other approach might be better suited for bosonic d.o.f as explored for NLSM model (see 2310.12512). Now extending the "CV" approach to SU(2) gauge theory [Kogut-Susskind Hamiltonian]



arXiv > quant-ph > arXiv:2310.12512

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Quantum Physics

[Submitted on 19 Oct 2023]

Continuous variable quantum computation of the $O(3)$ model in 1+1 dimensions

Raghav G. Jha, Felix Ringer, George Siopsis, Shane Thompson

We formulate the $O(3)$ non-linear sigma model in 1+1 dimensions as a limit of a three-component scalar field theory restricted to the unit sphere in the large squeezing limit. This allows us to describe the model in terms of the continuous variable (CV) approach to quantum computing. We construct the ground state and excited states using the coupled-cluster Ansatz and find excellent agreement with the exact diagonalization results for a small number of lattice sites. We then present the simulation protocol for the time evolution of the model using CV gates and obtain numerical results using a photonic quantum simulator. We expect that the methods developed in this work will be useful for exploring interesting dynamics for a wide class of sigma models and gauge theories, as well as for simulating scattering events on quantum hardware in the coming decades.

Comments: 28 pages, 16 figures
Subjects: **Quantum Physics (quant-ph)**; High Energy Physics - Lattice (hep-lat)
Cite as: arXiv:2310.12512 **[quant-ph]**
(or arXiv:2310.12512v1 **[quant-ph]** for this version)
https://doi.org/10.48550/arXiv.2310.12512

Submission history

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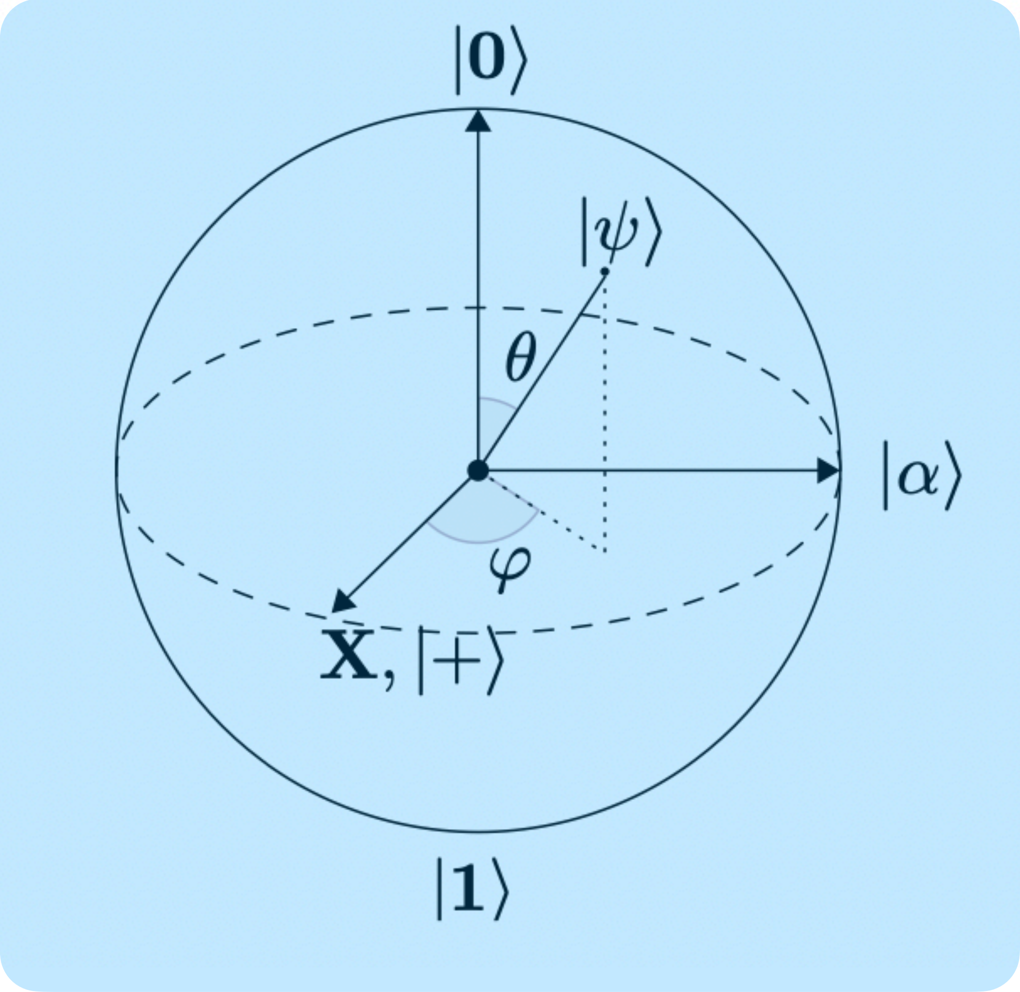
Qubits vs. Qumodes

	CV	Qubit
Basic element	Qumodes	Qubits
Relevant operators	Quadrature operators $\hat{x}, \hat{p}$ Mode operators $\hat{a}, \hat{a}^\dagger$	Pauli operators $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$
Common states	Coherent states $\lvert\alpha\rangle$ Squeezed states $\lvert z\rangle$ Number states $\lvert n\rangle$	Pauli eigenstates $\lvert 0/1\rangle, \lvert\pm\rangle, \lvert\pm i\rangle$
Common gates	Rotation, Displacement, Squeezing, Beamsplitter, Cubic Phase	Phase Shift, Hadamard, CNOT, T Gate

Quick Recap - Unitary gates

$$|+\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle + |1\rangle\Big)$$



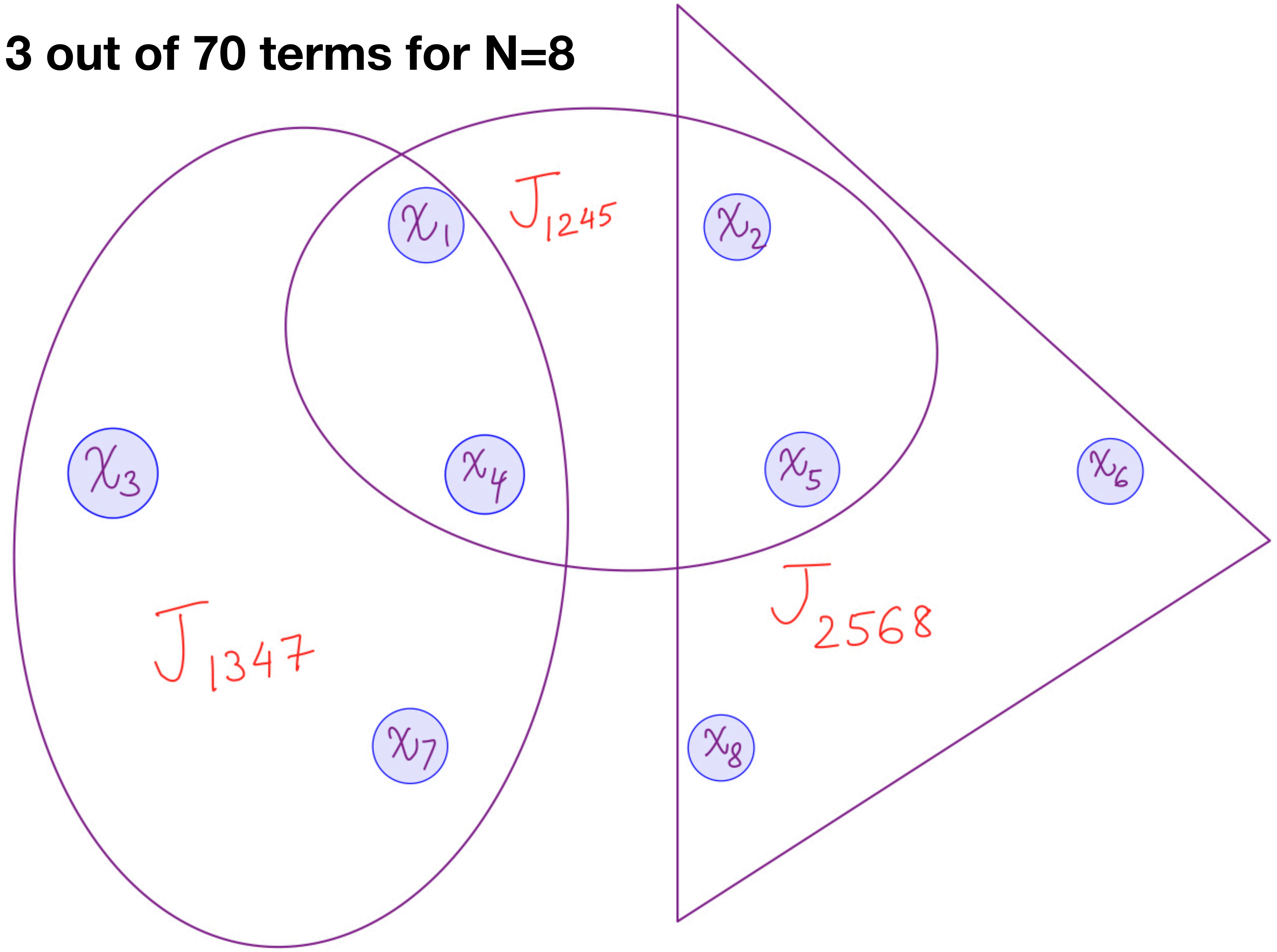
Operator	Gate(s)	Matrix
Pauli-X (X)	**X**	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
Pauli-Y (Y)	**Y**	$\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$
Pauli-Z (Z)	**Z**	$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$
Hadamard (H)	**H**	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$
Phase (S, P)	**S**	$\begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$
$\pi/8$ (T)	**T**	$\begin{bmatrix}1 & 0\\0 & e^{i\pi/4}\end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}$
Controlled Z (CZ)	**Z**	$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & -1\end{bmatrix}$
SWAP		$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 0 & 1 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{bmatrix}$

Questions?

SYK model

$$H = \frac{(i)^{q/2}}{q!} \sum_{i,j,k,\cdots,q=1}^{N} J_{ijk\cdots q} \quad \chi_i \chi_j \chi_k \cdots \chi_q,$$

3 out of 70 terms for N=8



- Model of N Majorana fermions with q-interaction terms with random coupling taken from a Gaussian distribution with $\overline{J_{\dots}} = 0,\ \overline{J^2_{\dots}} = \frac{q!J^2}{N^{q-1}}$.
- The fermions χ satisfy, $\chi_i\chi_j + \chi_j\chi_i = \delta_{ij}$. We will set $J = 1$. Note that it has units of energy and inverse time.
- In the limit of large number of fermions with $N \gg \beta J \gg 1$, the model has several interesting features such as maximal Lyapunov exponent.

Mapping fermions to qubits

$$\chi_{2k-1} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) X_k \mathbb{I}^{\otimes (N-2k)/2} \quad , \quad \chi_{2k} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) Y_k \mathbb{I}^{\otimes (N-2k)/2}$$

- N Majorana fermions requires N/2 qubits. We use the standard Jordan-Wigner mapping to write χ in terms of Pauli matrices X, Y, Z, and Identity.
- The SYK Hamiltonian is then written as sum of Pauli strings. The number of strings is $\binom{N}{q}$ and grows like $\sim N^q$. Simplest non-trivial case for is $N = q$ with one term. We restrict to $q = 4$.

Simplest case: N=4

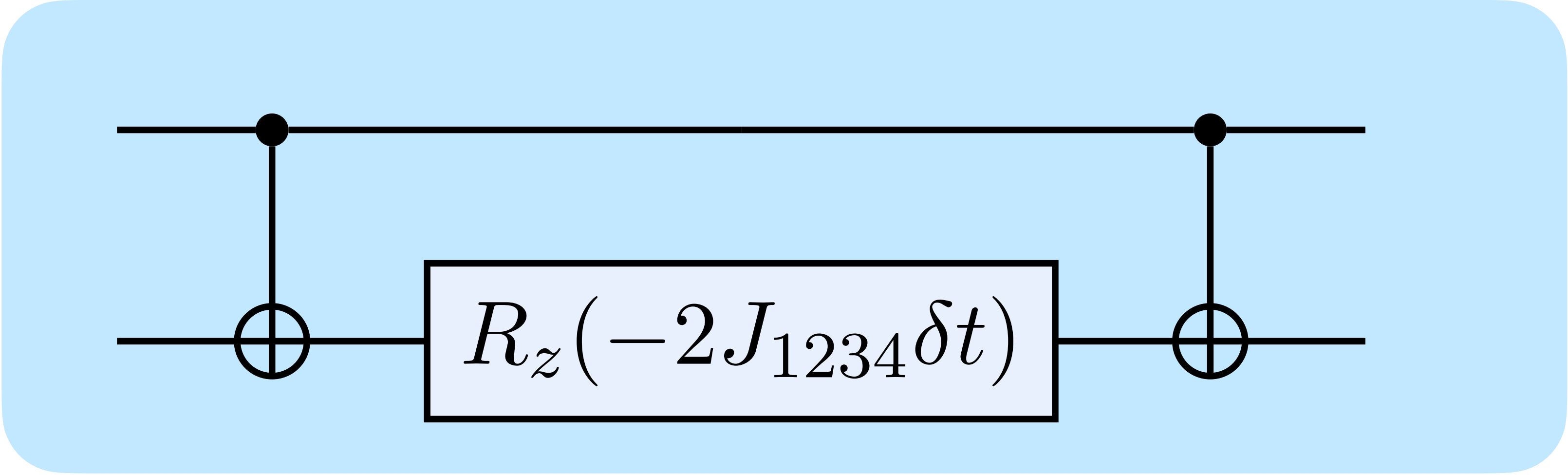
$$H = J_{1234}\chi_1\chi_2\chi_3\chi_4$$

$$\chi_1 = X\mathbb{1},\ \chi_2 = Y\mathbb{1},\ \chi_3 = ZX,\ \chi_4 = ZY$$

$$H = J_{1234}(X\mathbb{1})\cdot(Y\mathbb{1})\cdot(ZX)\cdot(ZY) = -J_{1234}ZZ$$

- The goal of quantum computation is to construct a unitary operator corresponding to this Hamiltonian. So, for this case we have $\exp(-iHt) = \exp(iJ_{1234}ZZt)$.

- This circuit is simple to construct and just needs 2 CNOTs and 1 rotation gate.



Circuit complexity

Definition: How many 2q-gates do we need to simulate the SYK model?

- Different approaches can be used to do the Hamiltonian simulation (aka time evolution). A popular method is Trotter method. It is based on Lie-Suzuki-Trotter product formula* (writing $H = \sum_{j=1}^{m} H_j$, $m \sim N^4$)

$$e^{-iHt} = \left(\prod_{j=1}^{m} e^{-iH_j t/r}\right)^r + \mathcal{O}\left(\sum_{j<k} \left|\left|[H_j, H_k]\right|\right| \frac{t^2}{r}\right),$$

- Depending on what error (ϵ) we desire in the time-evolution from the second term, we can compute the number of slices (r) we need to take. So, the complexity reduces to finding number of 2q-gates for each Trotter step. Recall that $N = 4$ needed just 2 2q-gates for each Trotter step.

* Corollary of Zassenhaus formula i.e., exp(t(X+Y)) = exp(tX) exp(tY) + O(t^2) (also known as dual of BCH formula).

Old work(s)

$\mathscr{C} = \mathcal{O}(N^{10}t^2/\epsilon)$	L. García-Álvarez et al., PRL 119, 040501 (2017)
$\mathscr{C} = \mathcal{O}(N^{8}t^2/\epsilon)$	Susskind, Swingle et al., arXiv: 2008.02303 (2020)
$\mathscr{C} = \tilde{\mathcal{O}}(N^{7/2}t)$	Babbush et al., Phys. Rev. A 99, 040301 (2019)

- The last one clearly is the most efficient, however, in the noisy-era implementing this is not feasible. It requires fault-tolerant quantum resources + ancillas since it is based on the basic idea of embedding H in a bigger vector space.

- Using the Trotter methods, the best seems to be $\sim N^8$. In our paper, we improved the complexity to $\mathscr{C} = \mathcal{O}(N^5t^2/\epsilon)$ which we now discuss.

Commuting terms

The costs can be simplified if we are little careful in splitting the SYK Hamiltonian.

The number of terms grows like $\sim N^4$, however, a large fraction of them commute with one another and can be collected together and then time-evolved more efficiently. We can find diagonalising circuit for each cluster and then apply time-evolution operator.

Finding optimal number of such clusters is a well-studied computer science problem. This is in general a NP-hard problem but various approx. algorithms exits.

Estimate based on general commutivity

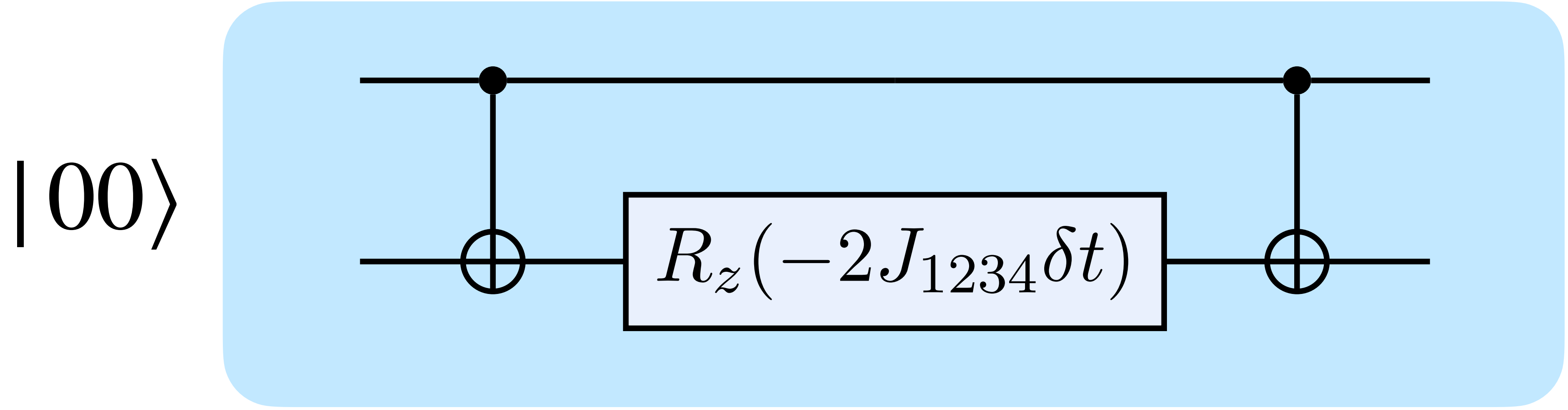
N	Pauli strings	Clusters	Two-qubit gates
4	1	1	2
6	15	5	30
8	70	6	110
10	210	23	498
12	495	57	1504
14	1001	92	3560
16	1820	116	6812
18	3060	175	11962
20	4845	246	19984

Return probability

- A simple observable we can compute is the probability that we return to the same initial state after some evolution time t i.e., $\mathscr{P}_0 = |\langle\psi_0|e^{-iHt}|\psi_0\rangle|^2$. For initial state, we take $|0\rangle^{\otimes N/2}$.

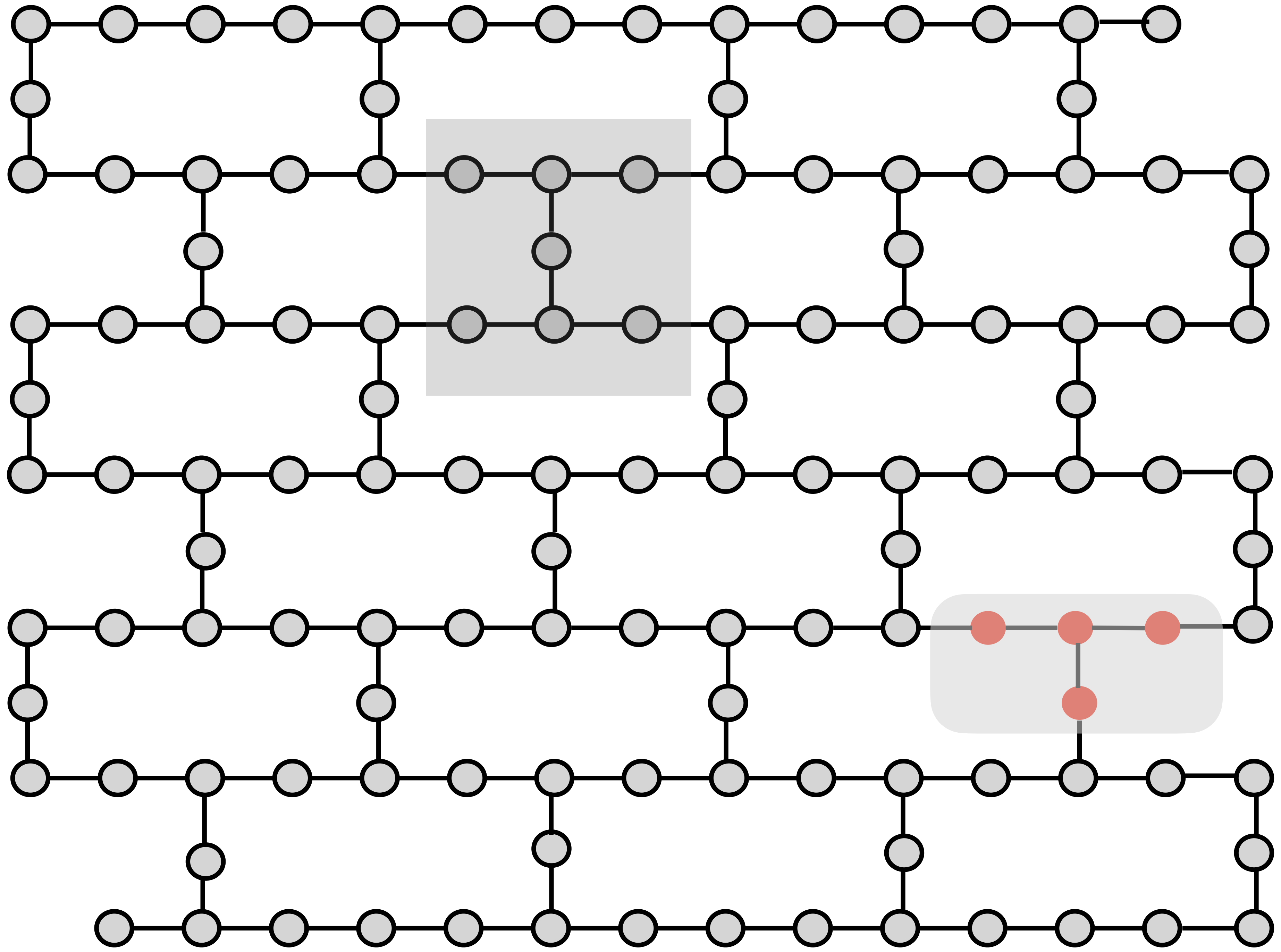
- For approximating the unitary, we use the first-order product formula and construct the corresponding quantum circuit.

- For $N=4$, we have a simple circuit of only two 2Q gates, so the entire circuit for return prob. is straightforward. For $N=6$, there are 30 2Q gates per step which we cannot show here.

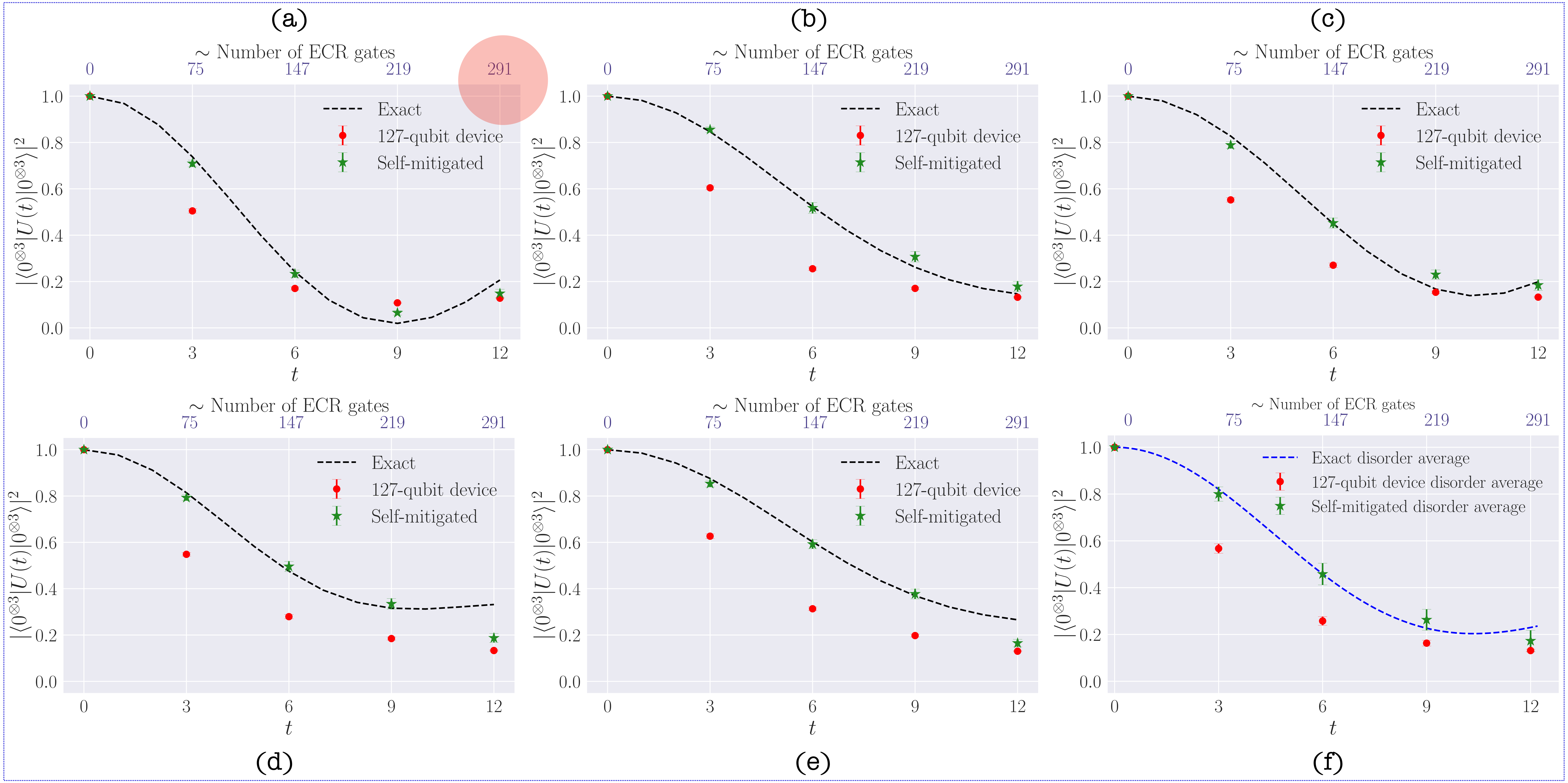


IBM chip topology

- We used the quantum computers available through IBM to simulate the SYK model. The topology of the processor is shown below. In practice, we need more gates than necessary. For example, we show a combination of qubits we used for $N = 8$. This chip topology is 'heavy-hex'.



Return probability - IBM Results



Error Mitigation

- The results from the 127-qubit device (red) agrees slightly less than those with self-mitigation (green). The red points have been read from some fixed number of measurements/shots and post-processed with mild mitigation including M3 to correct read-out errors and DD to increase coherence time of qubits.

- To get closer to the exact results, we found that an idea similar to CNOT only mitigation (known as self-mitigation) seems to help drastically. Basic idea introduced in Urbanek et al. arXiv: 2103.08591 and extended in Rahman et al. arXiv:2205.09247

M3 is a matrix measurement mitigation (MMM or M3) technique that solves for corrected measurement probabilities using a dimensionality reduction step

DD (dynamical decoupling) — a series of strong fast pulses are applied on the system which on average increases the lifetime of qubits and delays decoherence (or effect of interactions with environment)

SYK model - Bound on chaos

- SYK model famously saturated the Lyapunov exponent i.e., $\lambda = 2\pi T$ for $J/T \gg 1$ when N is large.
- One considers $C(t) = -\langle [W(t), V(0)]\,[W(t), V(0)] \rangle$ and the expansion of the commutator gives OTOC := $\langle W(t)V(0)W(t)V(0) \rangle_\beta = \mathrm{Tr}(\rho W(t)V(0)W(t)V(0))$ which characterizes quantum chaos.
- Suppose one starts at $t = 0$, and computes also the two-pt correlator given by $\langle W(t)W(0) \rangle$, the time scales at which the lower order correlators decay is called 'dissipation time'. After this time, the OTOC grows as $\exp(\lambda t)$ and saturates beyond $t_\star$ known as scrambling time. Black holes are fastest scramblers!
- These correlators have been computed up to $N = 60$ numerically i.e., H has ~million terms and matrix has size ~billion. Hard for classical computers.

Out-of-time correlators (OTOC)

- So the goal is to compute $\langle W(t)V(0)W(t)V(0)\rangle_\beta$ on a quantum computer. Thermal correlators are currently not easy to compute due to limited resources. One simplification we can make is consider the $\beta \to 0$ limit of OTOC. This is not at all interesting for holography, but this is where we must start. Hence, the density matrix is just $\rho \propto \mathbb{I}$.
- The unusual time-ordering of OTOC is also hard for quantum computers which often mean carrying out forward and backward evolution. We use a protocol (next slide) which uses only forward evolution to compute OTOC on quantum hardware.

Randomised Protocol

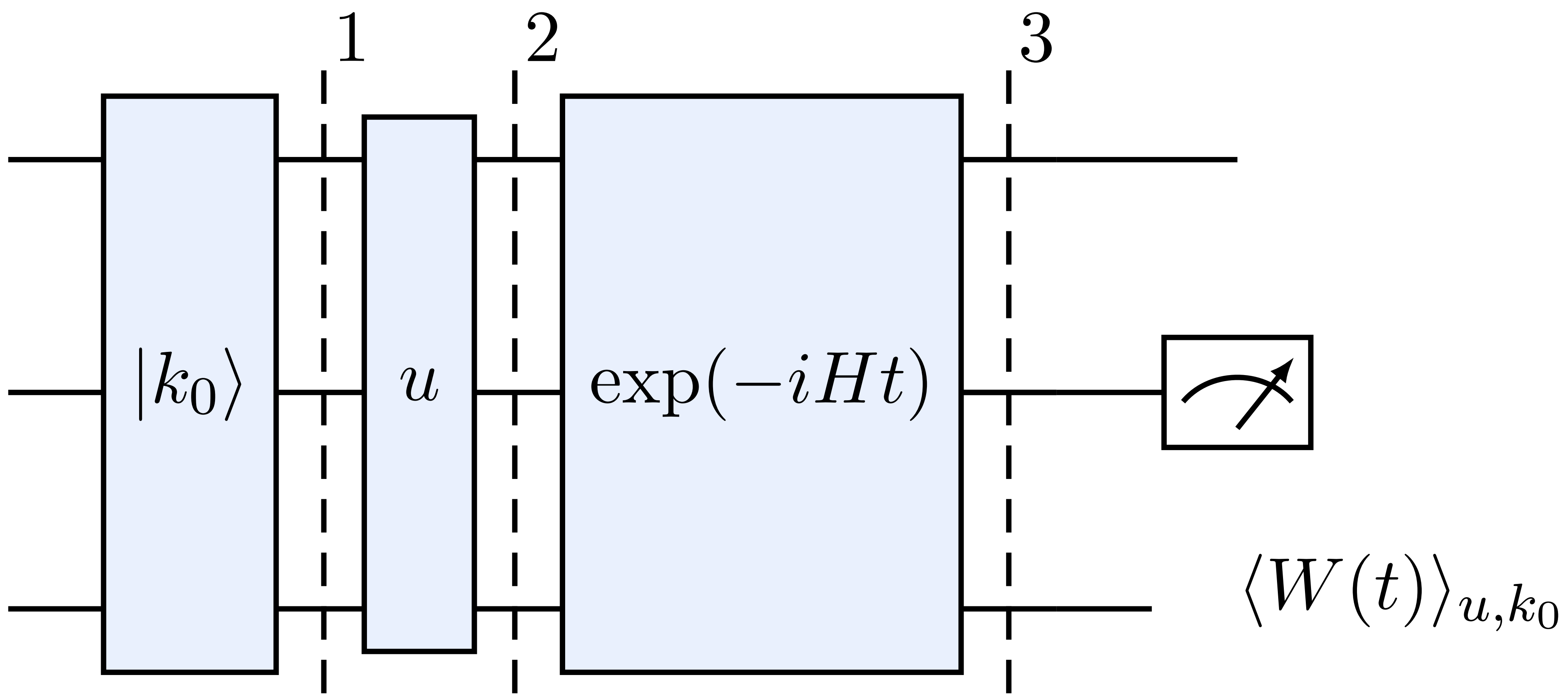
- There are various protocols to measure OTOC on quantum computers, see Swingle 2202.07060 for review.

- We use the one proposed in 1807.09087 now known as 'randomised protocol' since it computes OTOC through statistical correlations of observables measured on random states generated from a given matrix ensemble (CUE).

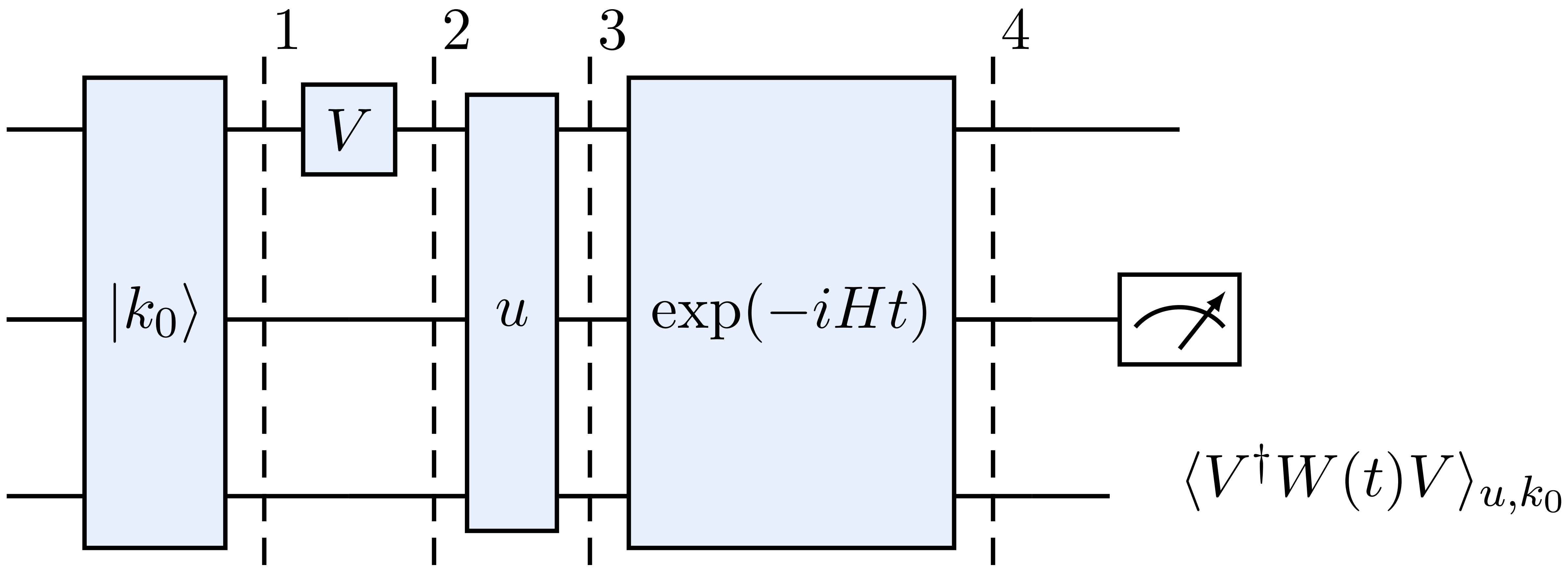
- Infinite-temp OTOC is given by Tr$(W(t)V^\dagger W(t)V) \propto \overline{\langle W(t)\rangle_u \langle V^\dagger W(t) V\rangle_u}$ where the average is over different random states $|\psi_u\rangle$ prepared by acting with random unitary on arbitrary state say $|0\rangle^{\otimes n}$. Note that this protocol works when W is traceless operator.

Randomised Protocol

- We need two measurements (between which we compute the statistical correlation) and it is shown below. This is the global version of the protocol (since u has support over all qubits). There is also a local version of the protocol. Note that cost of decomposing arbitrary u increases exponentially, one can instead use unitary from a subset of Haar measure. They are called unitary t-designs* in literature.



(a)

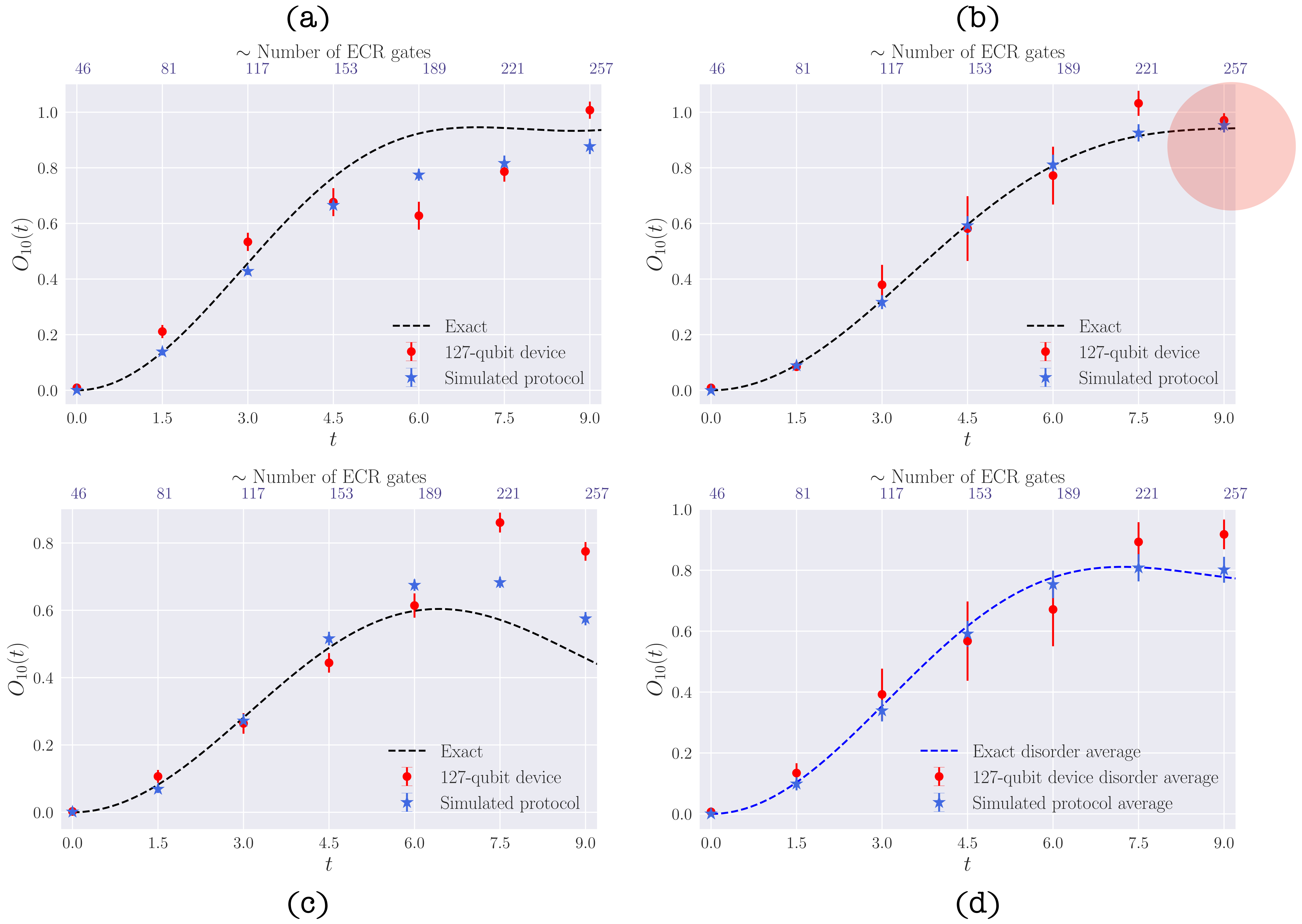


(b)

t-designs equivalent to first t moments of Haar group

OTOC Results

- We used ibm_cusco and ibm_nazca to obtain the results show for $N = 6$. We took simplest operators where both W and V were taken to be single Pauli. We see good agreement without need to do self-mitigation like we did for return probability.



Finite-temperature SYK model

- We considered OTOC measured over random states (maximally mixed) generated i.e., $\beta = 1/T = 0$. However, much of interesting Physics of the SYK happens in the region $\beta \gg 1$ and classical computations have argued that you need $\beta \sim 70$ to extract Lyapunov exponents close to the chaos bound.

- Finite-temperature OTOCs are difficult for quantum computers in general. No simple/general cost-effective protocol exists. To move towards this goal, we are studying the preparation of Gibbs (thermal) states on quantum computer for the SYK model.

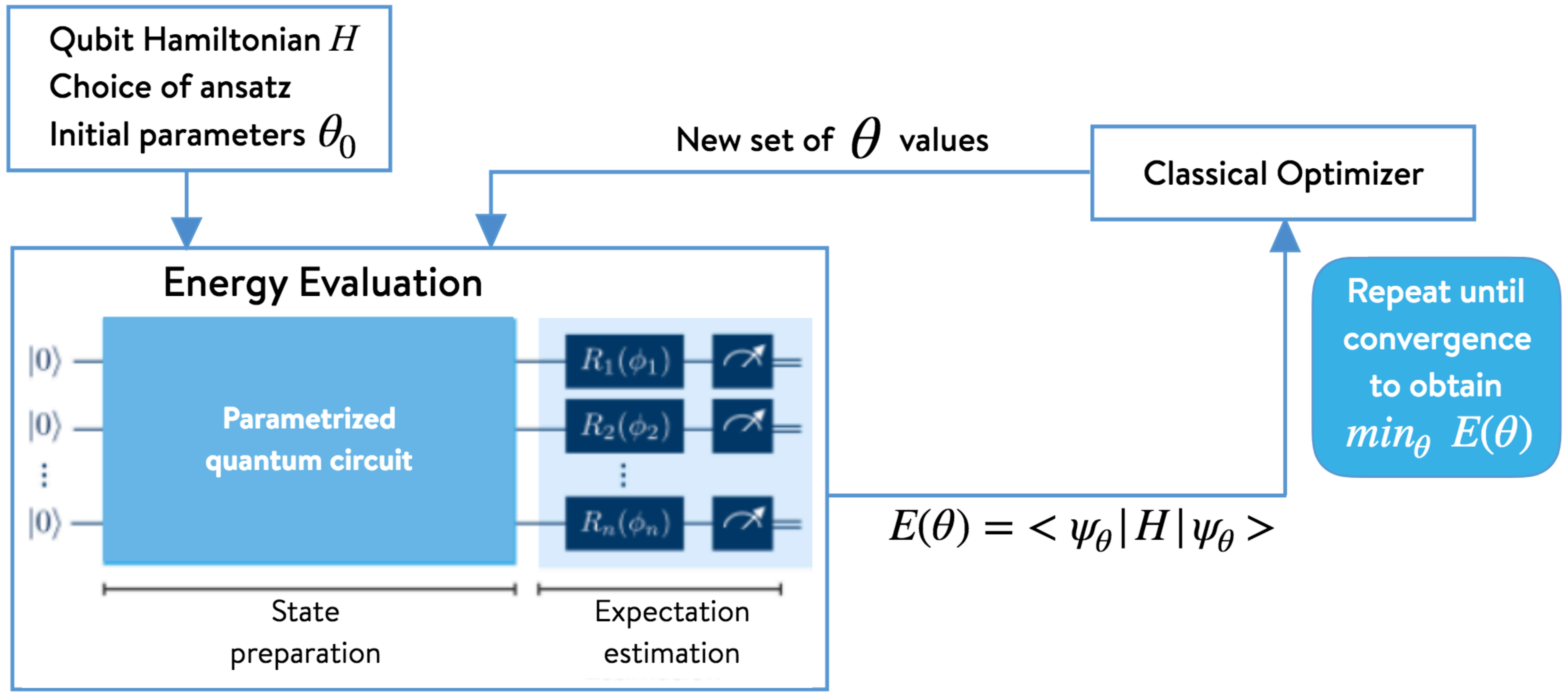
- In addition to the thermal state (mixed) of the SYK model, one can also consider a purification of this known as thermo-field double state (TFD). TFD state is a pure state (up to unitary trans.) of some other system (for ex: coupled SYK model) and when we perform partial trace over either system, we recover thermal state on the other one.

VQE algorithm

- Before we move to preparation of Gibbs state, let's us look at popular algorithm for preparing (approximate) ground states on QC.

- Hybrid classical/quantum algorithm to find the ground state problem of a given Hamiltonian by finding the parameters of a quantum circuit ansatz that minimizes the Hamiltonian expectation value.

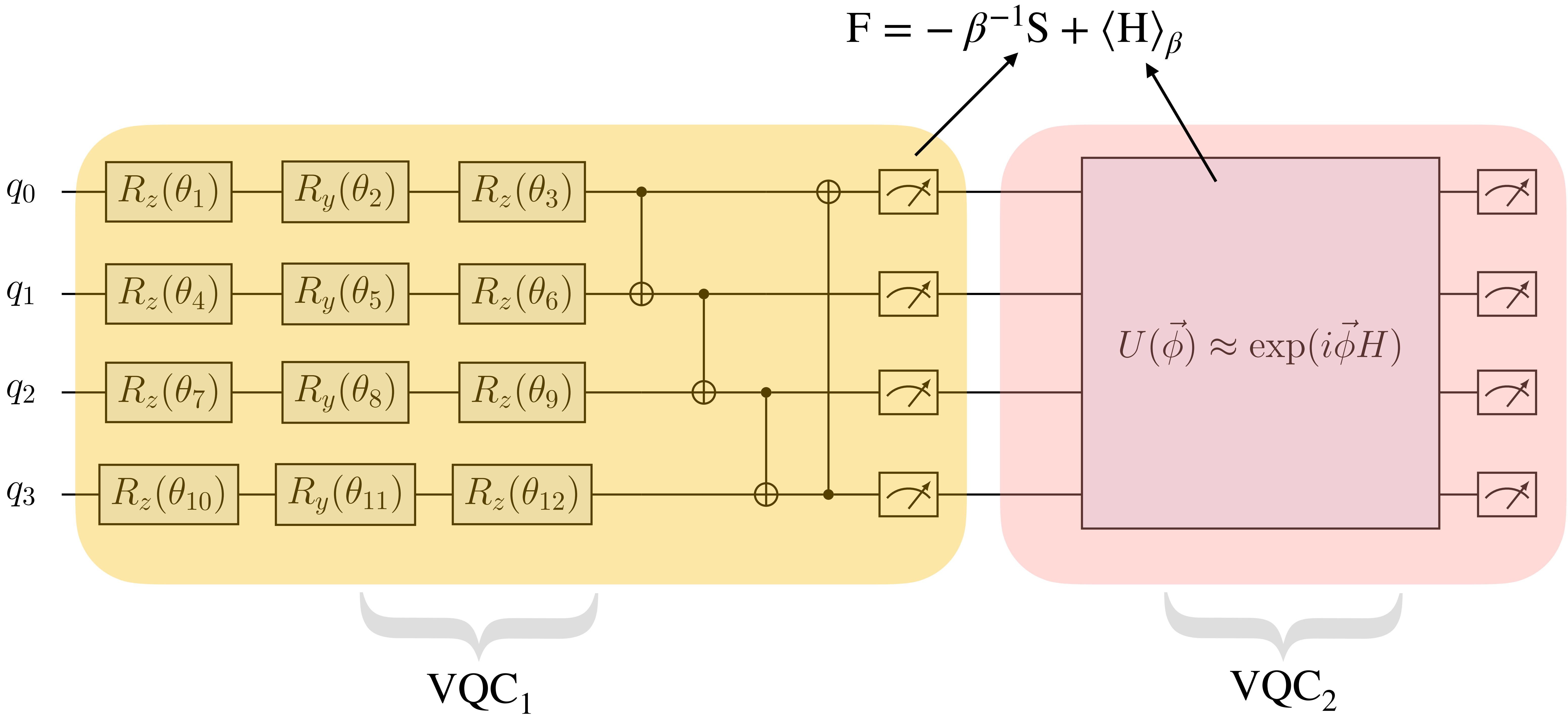
- It primarily consists of three steps: 1) Prepare initial ansatz on QC i.e., $|\psi(\vec{\Theta})\rangle$, 2) Measure energy on QC and optimise the parameters Θ using classical optimisers and 3) Repeat until desired convergence is achieved.



VQE for finite temperatures

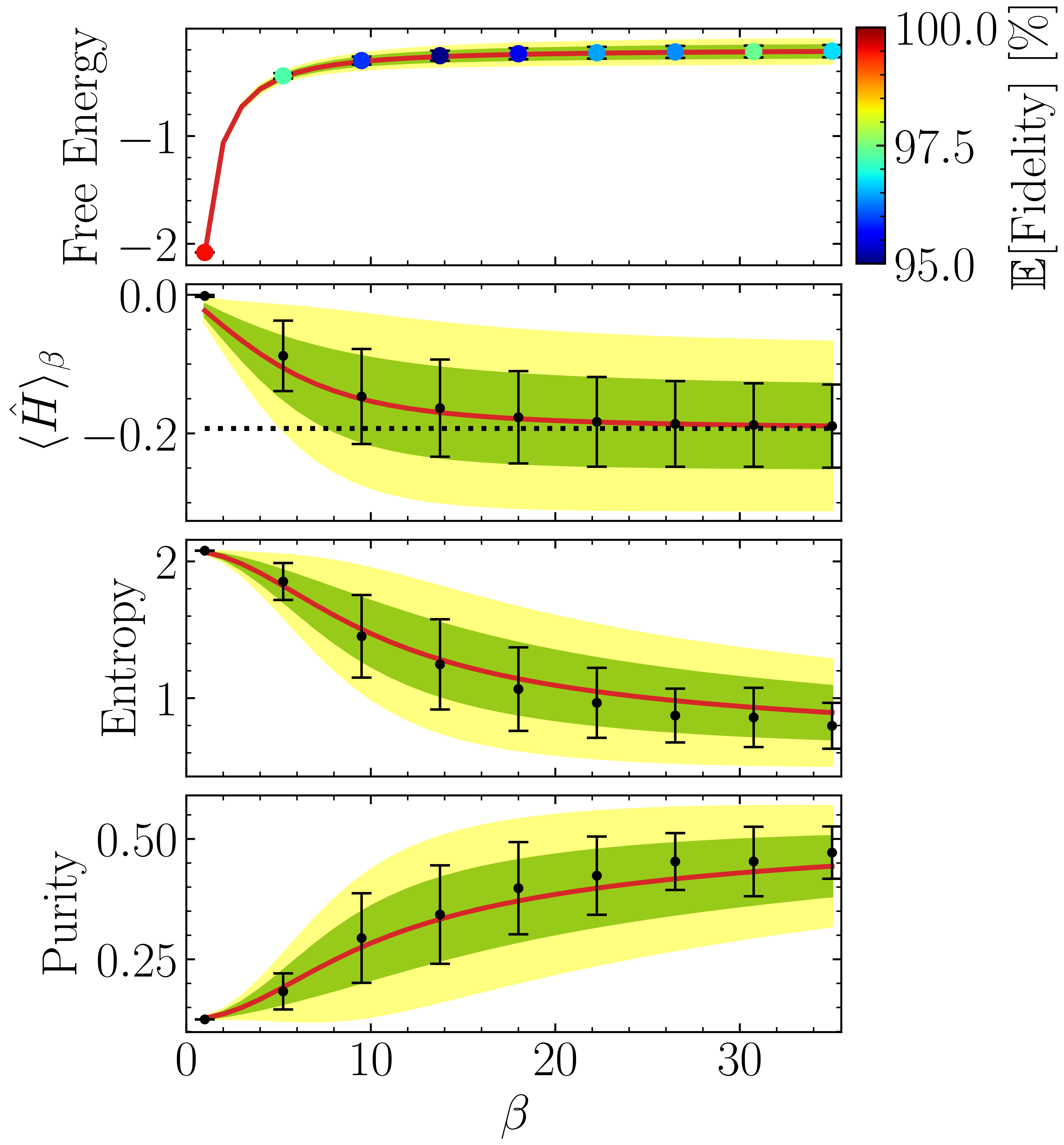
arXiv: 2406.15545

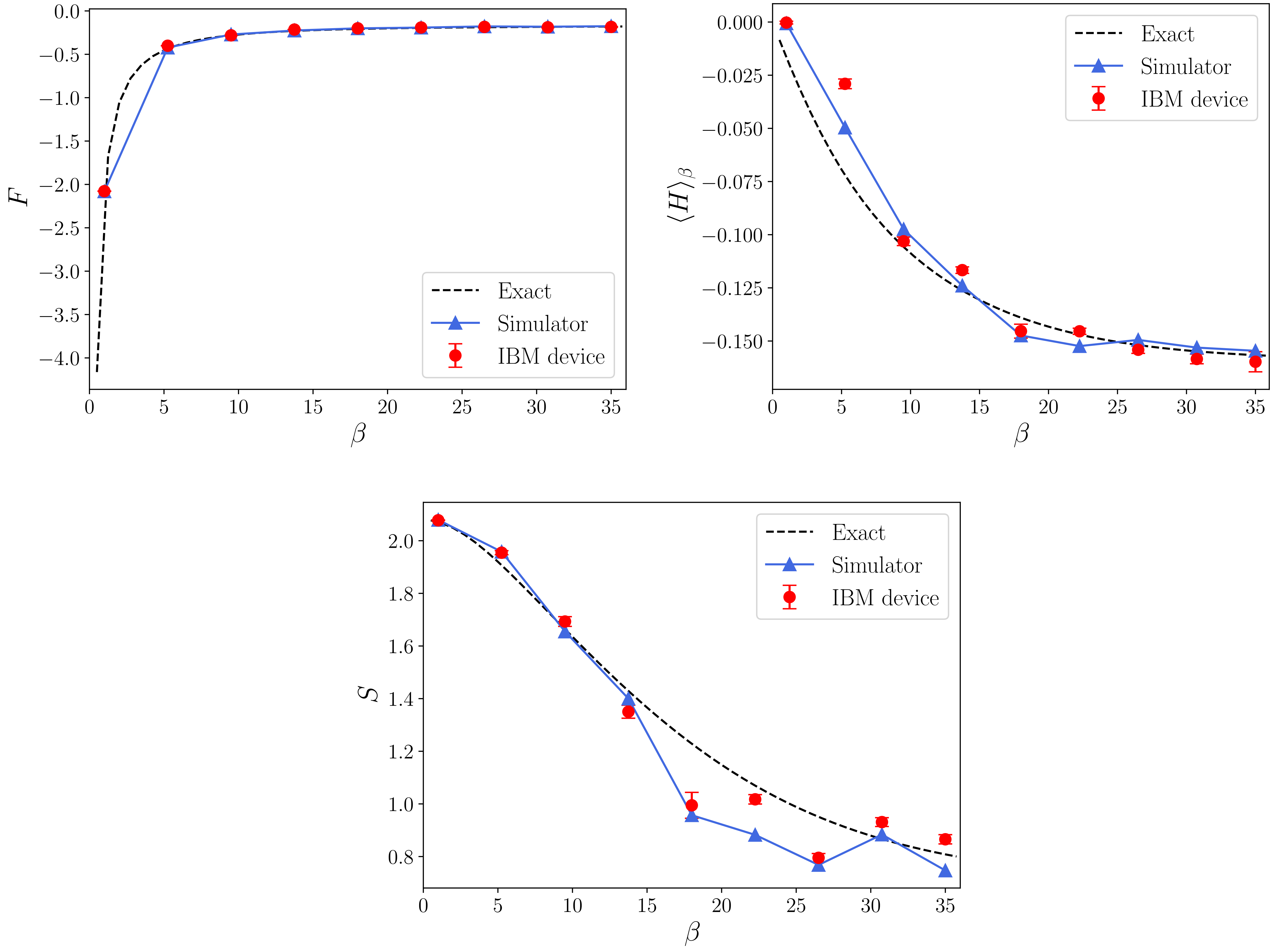
- Finite-temperature VQE methods are still an active area of research. Many proposals exist. The cost function is no longer E but rather $E - TS$ (free energy) which can be hard to compute on QC.



Selisko et al., 2208.07621

Finite-temperature SYK model



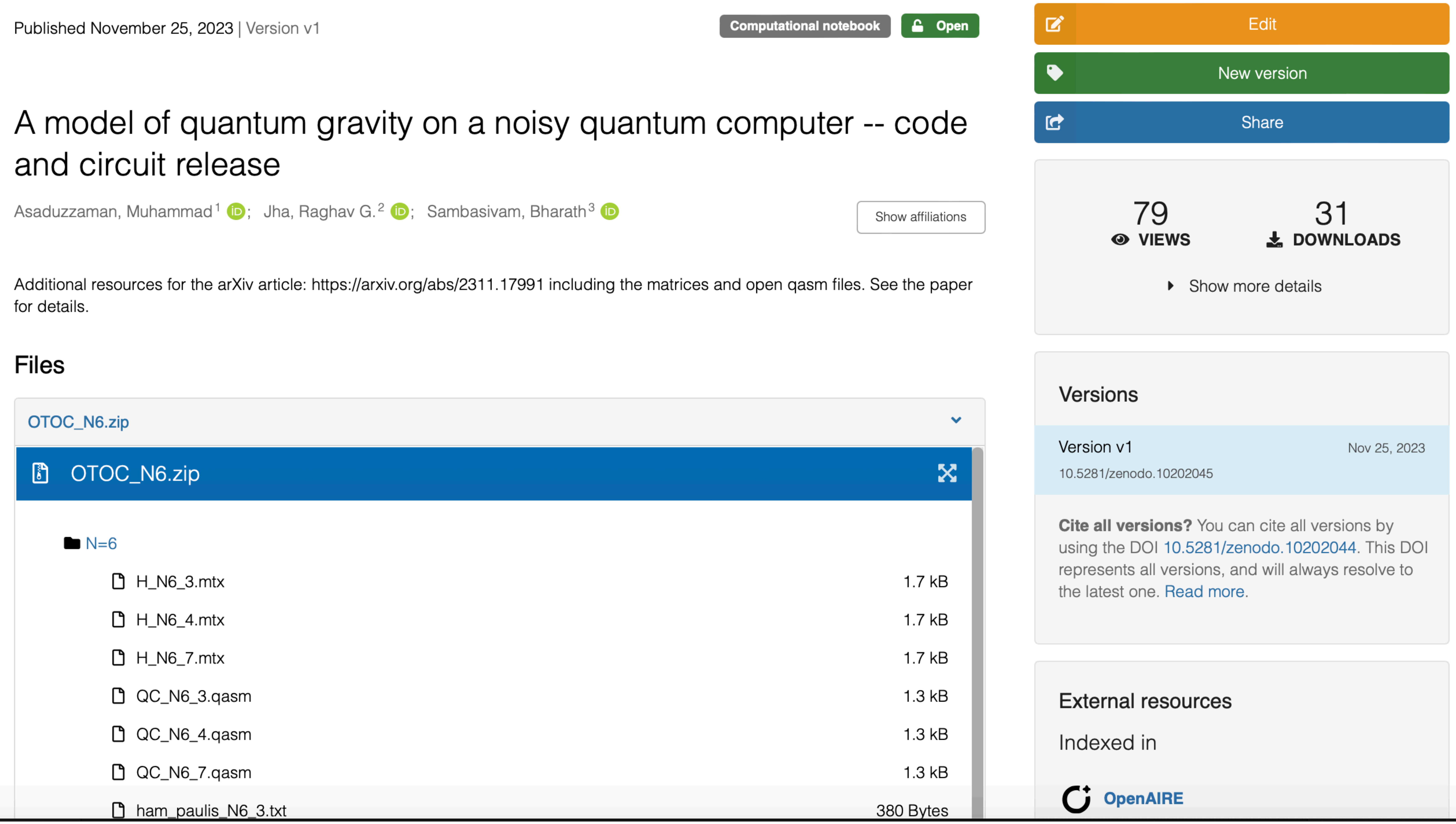


Results from PennyLane simulator

Summary

- We are entering an era where we can do small computations reliably on quantum computers. Exploring these toy models will hopefully reveal to us better algorithms/methods.
- It is important to note that if we can model the noise in these quantum devices, we can mitigate and get reasonable results!
- Preparation of thermal states and its purifications known as TFD states is still challenging and search for better ways to do this (by minimizing over gate costs and improving fidelity) is interesting research direction.

Resources and Data Statement



Published November 25, 2023 | Version v1

Computational notebook | Open

A model of quantum gravity on a noisy quantum computer -- code and circuit release

Asaduzzaman, Muhammad[1]; Jha, Raghav G.[2]; Sambasivam, Bharath[3]

Show affiliations

Additional resources for the arXiv article: https://arxiv.org/abs/2311.17991 including the matrices and open qasm files. See the paper for details.

Files

OTOC_N6.zip

OTOC_N6.zip

- N=6
  - H_N6_3.mtx 1.7 kB
  - H_N6_4.mtx 1.7 kB
  - H_N6_7.mtx 1.7 kB
  - QC_N6_3.qasm 1.3 kB
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External resources

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Both classical and quantum code available at: https://github.com/rgjha/SYKquantumcomp

Thank you