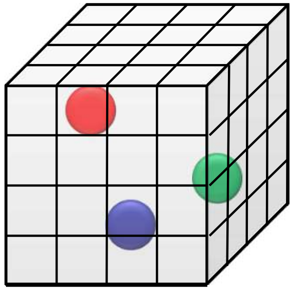
Toward Lattice Gauge Theory on Quantum Computers

Arata Yamamoto (University of Tokyo)

Self-Introduction

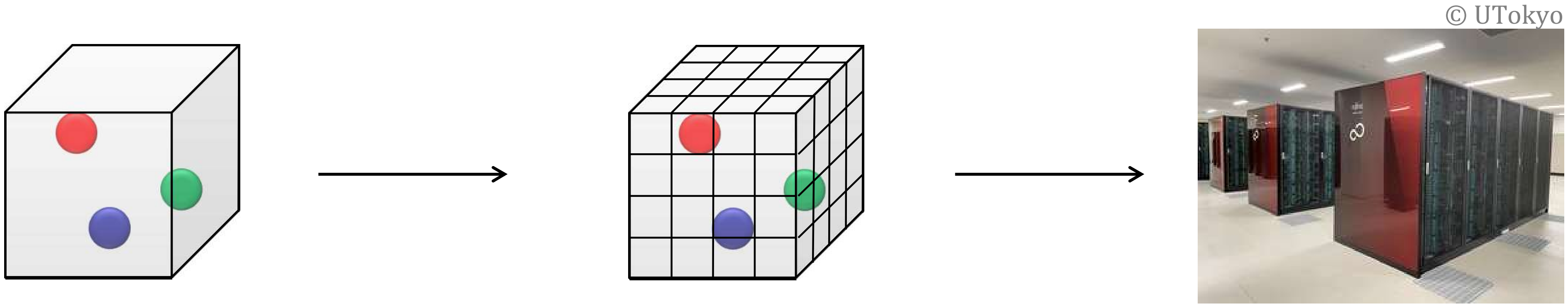
affiliation : **Hongo campus, The University of Tokyo**



field: **hadron theory**

research: **lattice gauge theory / lattice QCD**

Introduction



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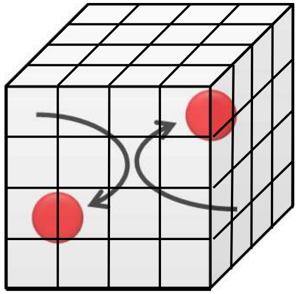
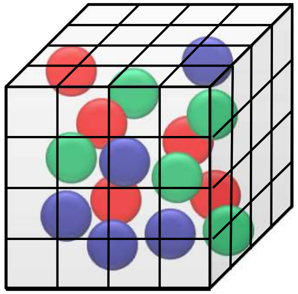
gauge theory → lattice gauge theory → supercomputer

Introduction

open problems in lattice gauge theory



non-equilibrium



fermionic matter

tensor network

© IBM

quantum computer

Introduction

open problems in lattice gauge theory

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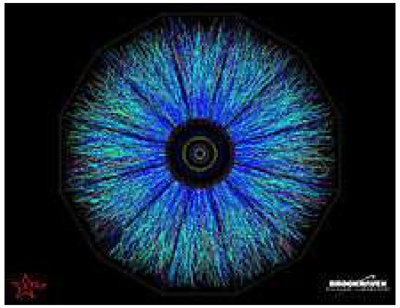
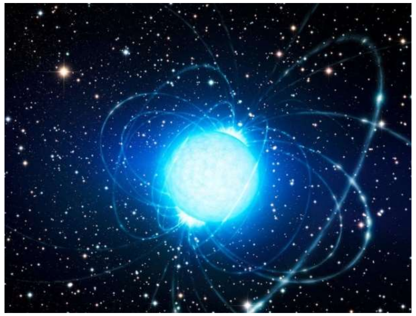


non-equilibrium

https://en.wikipedia.org/wiki/Magnetar



fermionic matter

tensor network

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quantum computer

Contents

2. Quantum computer

Quantum computer

classical computer

© UTokyo

quantum computer

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c-bit $=0$ or 1

q-bit $= a|0\rangle + b|1\rangle$

operation $= \{+, -, \times, \cdots\}$

$$\text{operation} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix}$$

N bits $= N$ c-numbers

N bits $=$ superposition of 2^N states

Quantum computer

noisy intermediate-scale quantum (NISQ)

Quantum computer

noisy intermediate-scale quantum (NISQ)

limited number of qubits

$$100\ \text{qubits} = 8\ \text{bytes(complex)} \times 2^{100} \sim 10^{16}\ \text{Pbytes}$$
$$\gg \text{classical memory}$$

Quantum computer

noisy intermediate-scale quantum (NISQ)

limited number of qubits

$$100\ \text{qubits} \;=\; 8\ \text{bytes(complex)} \times 2^{100} \;\sim\; 10^{16}\ \text{Pbytes}$$

$$\gg\ \text{classical memory}$$

device error

we need “error mitigation”

Quantum computer

quantum device roadmap

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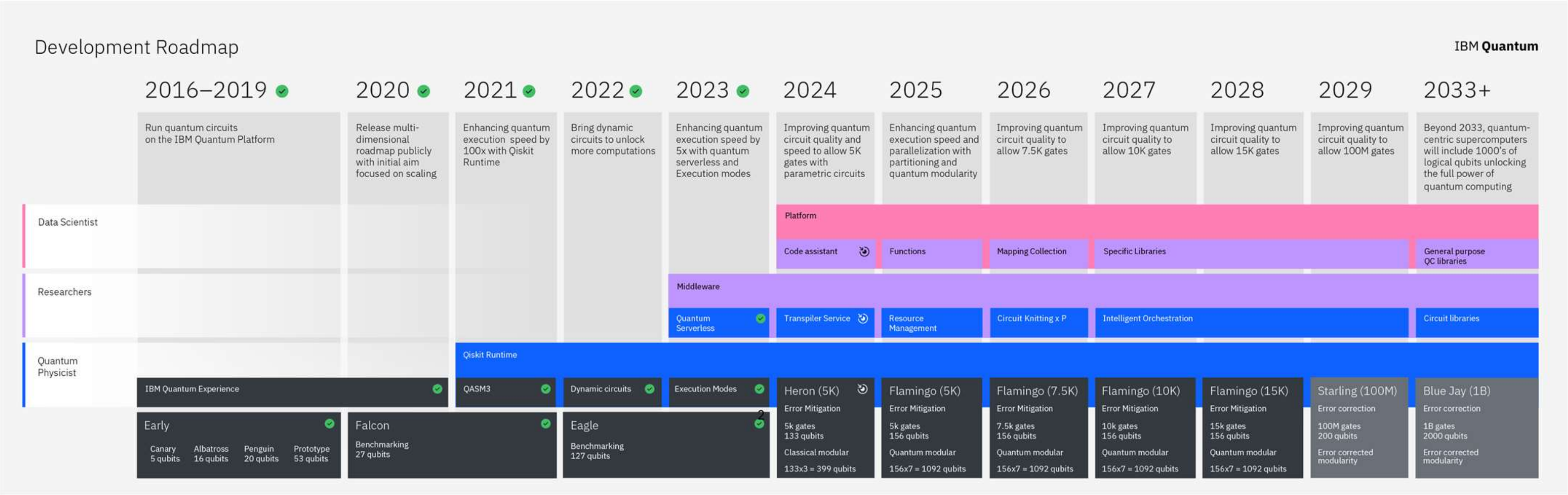


Development Roadmap

IBM **Quantum**

2016–2019 ✓
Run quantum circuits on the IBM Quantum Platform

2020 ✓
Release multi-dimensional roadmap publicly with initial aim focused on scaling

2021 ✓
Enhancing quantum execution speed by 100x with Qiskit Runtime

2022 ✓
Bring dynamic circuits to unlock more computations

2023 ✓
Enhancing quantum execution speed by 5x with quantum serverless and Execution modes

2024
Improving quantum circuit quality and speed to allow 5K gates with parametric circuits

2025
Enhancing quantum execution speed and parallelization with partitioning and quantum modularity

2026
Improving quantum circuit quality to allow 7.5K gates

2027
Improving quantum circuit quality to allow 10K gates

2028
Improving quantum circuit quality to allow 15K gates

2029
Improving quantum circuit quality to allow 100M gates

2033+
Beyond 2033, quantum-centric supercomputers will include 1000's of logical qubits unlocking the full power of quantum computing

Data Scientist
- Platform
  - Code assistant
  - Functions
  - Mapping Collection
  - Specific Libraries
  - General purpose QC libraries

Researchers
- Middleware
  - Quantum Serverless ✓
  - Transpiler Service
  - Resource Management
  - Circuit Knitting x P
  - Intelligent Orchestration
  - Circuit libraries

Quantum Physicist
- IBM Quantum Experience ✓
- Qiskit Runtime
  - QASM3 ✓
  - Dynamic circuits ✓
  - Execution Modes ✓
- Early ✓
  - Canary 5 qubits
  - Albatross 16 qubits
  - Penguin 20 qubits
  - Prototype 53 qubits
- Falcon ✓
  - Benchmarking 27 qubits
- Eagle ✓
  - Benchmarking 127 qubits
- Heron (5K)
  - Error Mitigation
  - 5k gates 133 qubits
  - Classical modular
  - 133x3 = 399 qubits
- Flamingo (5K)
  - Error Mitigation
  - 5k gates 156 qubits
  - Quantum modular
  - 156x7 = 1092 qubits
- Flamingo (7.5K)
  - Error Mitigation
  - 7.5k gates 156 qubits
  - Quantum modular
  - 156x7 = 1092 qubits
- Flamingo (10K)
  - Error Mitigation
  - 10k gates 156 qubits
  - Quantum modular
  - 156x7 = 1092 qubits
- Flamingo (15K)
  - Error Mitigation
  - 15k gates 156 qubits
  - Quantum modular
  - 156x7 = 1092 qubits
- Starling (100M)
  - Error correction
  - 100M gates 200 qubits
  - Error corrected modularity
- Blue Jay (1B)
  - Error correction
  - 1B gates 2000 qubits
  - Error corrected modularity

error mitigation

error correction

Quantum computer

quantum simulation roadmap

2024

- ✓ NISQ or emulator
- ✓ benchmark test
- ✓ usage & algorithm

2029 ~

- ✓ small-scale QC
- ✓ toy model

20XX ~

- ✓ large-scale QC
- ✓ realistic theory

3. Lattice gauge theory

Lattice gauge theory

path integral formalism

$$Z = \int d\Psi \, e^{-S}$$

c-number classical action

for classical computing

Hamiltonian formalism

$$E = \langle \Psi | H | \Psi \rangle$$

Hamiltonian operator quantum state

for quantum computing

Lattice gauge theory

fermion (electron, quark, etc.)

$$|\psi\rangle = a\underset{\substack{\uparrow \\ \text{“empty”}}}{|0\rangle} + b\underset{\substack{\uparrow \\ \text{“occupied”}}}{|1\rangle} \qquad \leftrightarrow \qquad \text{1 qubit}$$

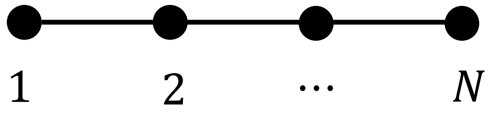
Lattice gauge theory

fermion (electron, quark, etc.)

$$|\psi\rangle = a\underset{\substack{\uparrow \\ \text{“empty”}}}{|0\rangle} + b\underset{\substack{\uparrow \\ \text{“occupied”}}}{|1\rangle} \qquad \leftrightarrow \qquad \text{1 qubit}$$

total state vector $$|\Psi\rangle = \prod_{x=1}^{N} |\psi(x)\rangle \qquad \leftrightarrow \qquad \text{N qubit}$$

total dimension $$D = 2^N$$



Lattice gauge theory

gauge field (photon, gluon, etc.)

Z_2 gauge theory $|g\rangle = c_0|+1\rangle + c_1|-1\rangle$ ↔ 1 qubit

total state vector $|\Psi\rangle = \prod_{x=1}^{N-1} |g(x)\rangle$ ↔ N − 1 qubit

total dimension $D = 2^{N-1}$

Lattice gauge theory

gauge field (photon, gluon, etc.)

Z_2 $\quad |g\rangle = c_0|+1\rangle + c_1|-1\rangle$ ↔ 1 qubit

Z_4 $\quad |g\rangle = c_0|e^{i0}\rangle + c_1|e^{i\pi/2}\rangle + c_2|e^{i\pi}\rangle + c_3|e^{i3\pi/2}\rangle$ ↔ 2 qubits

$\vdots$

Z_n $\quad |g\rangle = c_0|e^{i0}\rangle + c_1|e^{i2\pi/n}\rangle + \cdots + c_{n-1}|e^{i2\pi(n-1)/n}\rangle$ ↔ $\log_2 n$ qubits

$\xrightarrow[n \to \infty]{} \quad U(1)$ ↔ ∞ qubits

Lattice gauge theory

time evolution

$$|\Psi'\rangle = U|\Psi\rangle = e^{-iHt}|\Psi\rangle$$

classical simulation

$$\begin{pmatrix} c'_1 \\ \vdots \\ c'_D \end{pmatrix} = \begin{pmatrix} U_{11} & \cdots & U_{1D} \\ \vdots & \ddots & \vdots \\ U_{D1} & \cdots & U_{DD} \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_D \end{pmatrix} \Bigg\} \quad D = O(2^N) \text{ complex numbers}$$

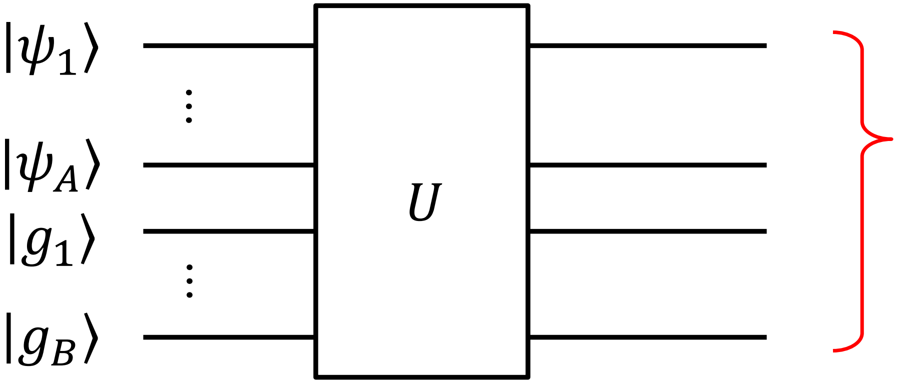
simulation cost $D^2 = O(4^N)$

Lattice gauge theory

time evolution

$$|\Psi'\rangle = U|\Psi\rangle = e^{-iHt}|\Psi\rangle$$

quantum simulation



$O(N)$ qubits

simulation cost $O(N)$

Lattice gauge theory

Gauss law constraint

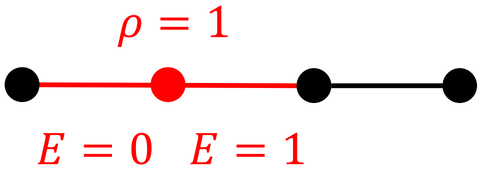
$$\nabla \cdot \underset{\substack{\uparrow \\ \text{electric field}}}{\vec{E}}(x) = \underset{\substack{\uparrow \\ \text{charge density}}}{\rho}(x)$$

Lattice gauge theory

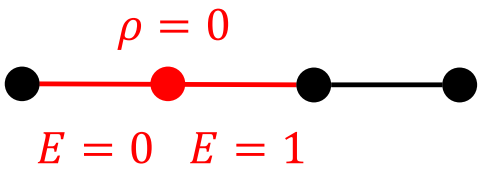
Gauss law constraint

$$\underset{\substack{\uparrow\\ \text{electric field}}}{\nabla\cdot\vec{E}(x)} = \underset{\substack{\uparrow\\ \text{charge density}}}{\rho(x)}$$

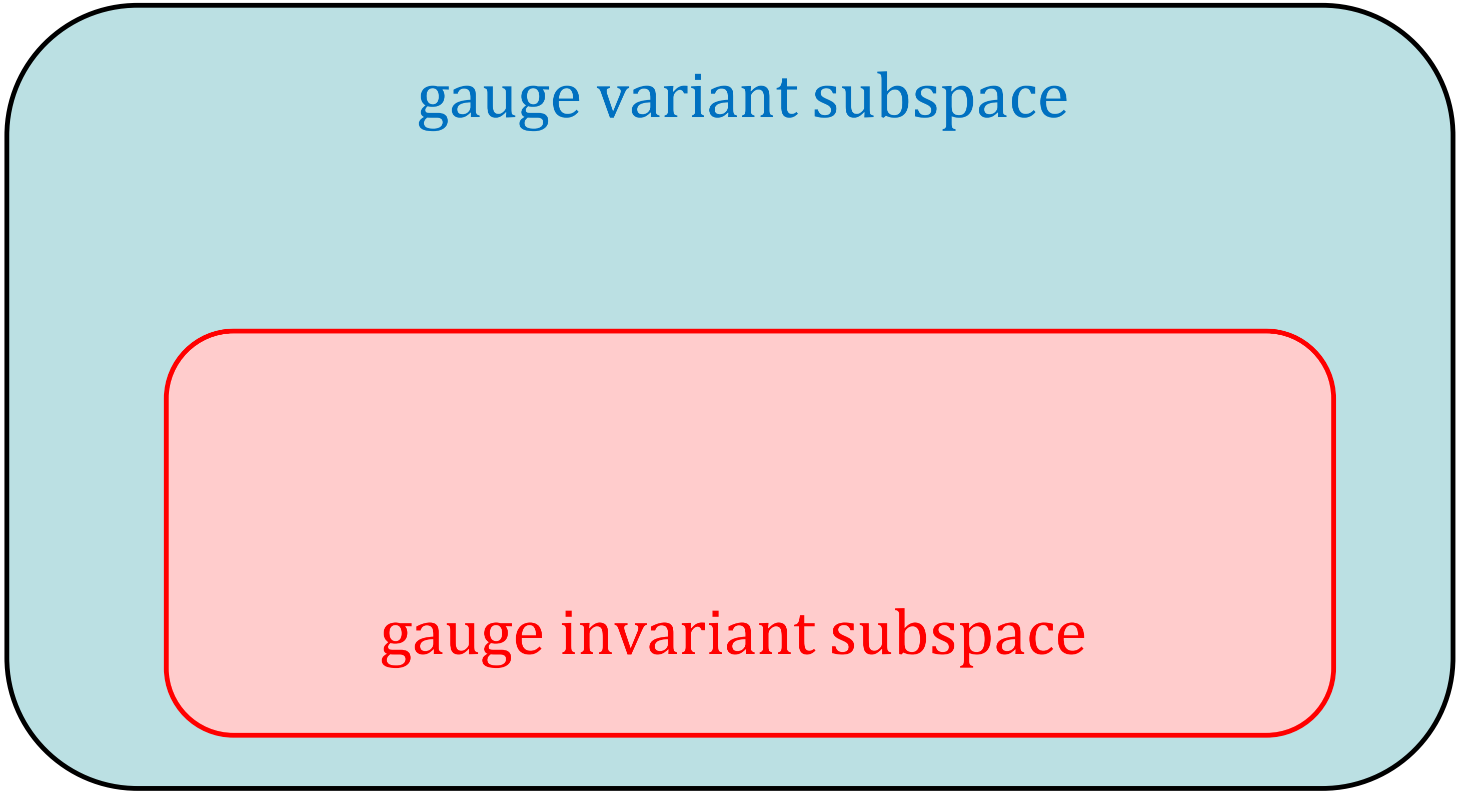
gauge invariant state: $\nabla\cdot\vec{E}(x)|\Psi\rangle = \rho(x)|\Psi\rangle$



gauge variant state: $\nabla\cdot\vec{E}(x)|\Psi\rangle \neq \rho(x)|\Psi\rangle$



Lattice gauge theory

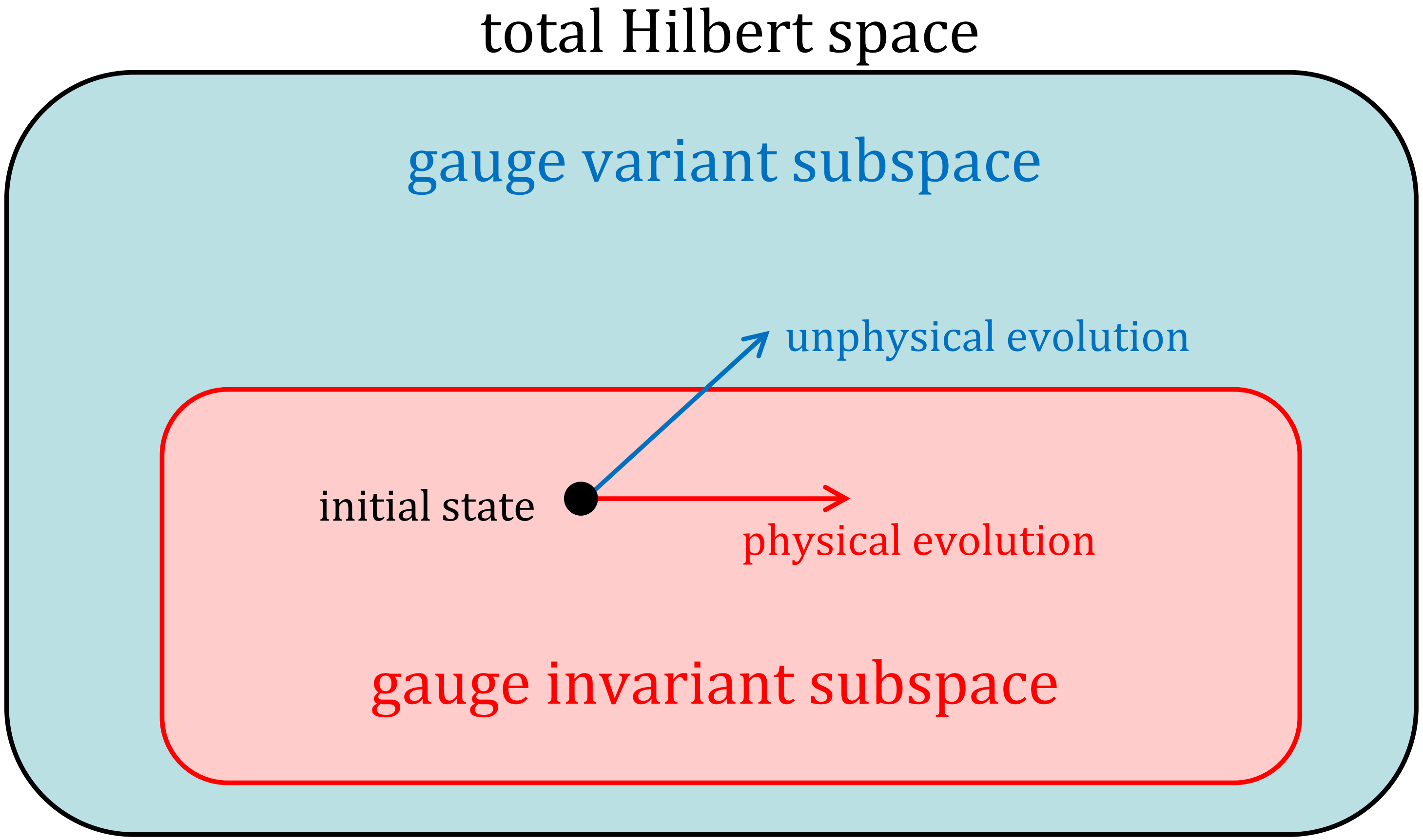


total Hilbert space

gauge variant subspace

gauge invariant subspace

Lattice gauge theory



total Hilbert space

gauge variant subspace

unphysical evolution

initial state

physical evolution

gauge invariant subspace

4. Examples

Examples

what to do:

- ✓ error robust algorithm
- ✓ resource efficient algorithm

- ✓ qubit encoding of SU(3)
- ✓ continuum limit

specific problems in lattice gauge theory

(skipped in this talk)

toy models:

- ✓ Z_n gauge theory
- ✓ 1-dim. gauge theory

Examples

what to do:

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specific problems in lattice gauge theory

(skipped in this talk)

toy models:

- ✓ Z_n gauge theory

explained here

- ✓ 1-dim. gauge theory

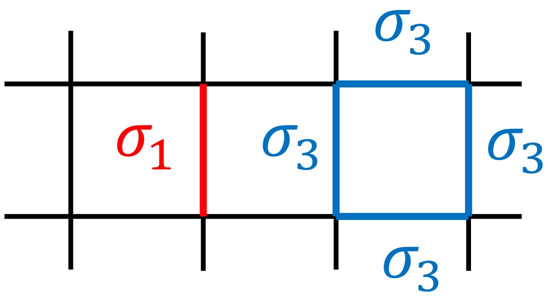
Examples

2D Z_2 pure gauge theory

$$H = -\sum_{\text{link}} \sigma_1 - \sum_{\text{plaq}} \sigma_3\sigma_3\sigma_3\sigma_3$$

↑ electric field

↑ magnetic field



Examples

2D Z_2 pure gauge theory

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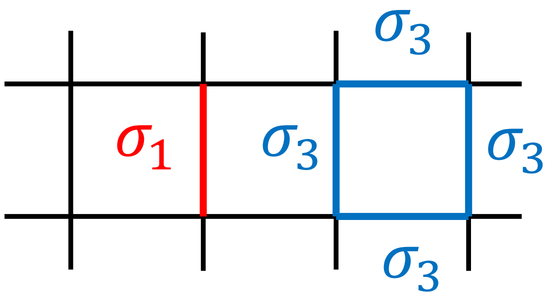
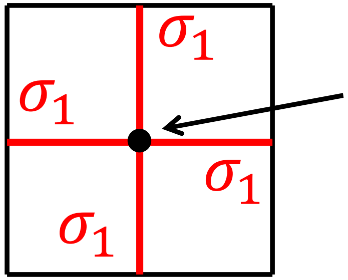


Gauss law

$$\sigma_1\sigma_1\sigma_1\sigma_1 = (-1)^{\rho(x)}$$

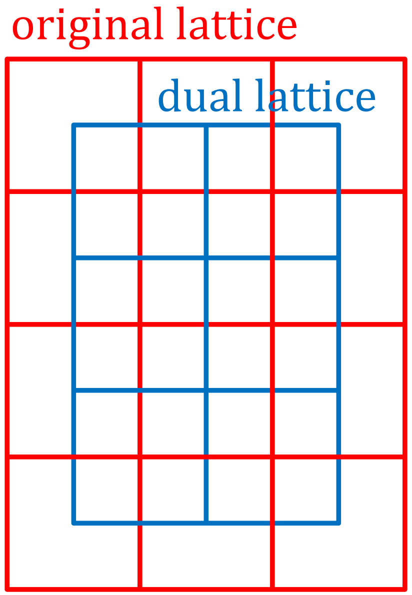


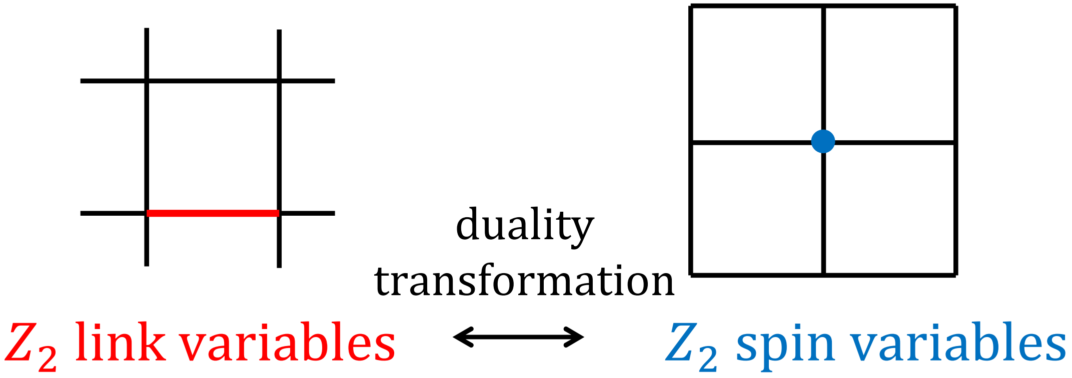
static charge

$\rho = 0$ or 1

Examples

Wegner duality Wegner (1971)





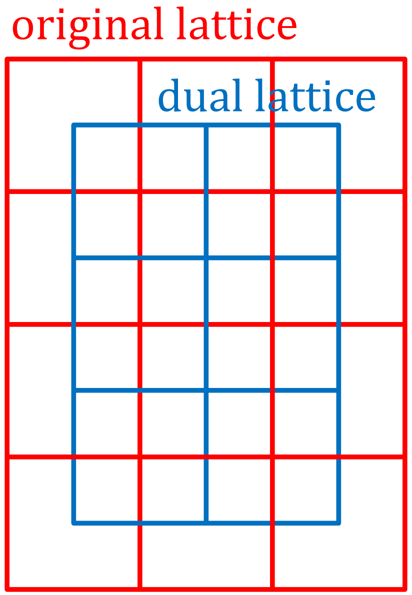
Z_2 link variables

duality
transformation
⟷

Z_2 spin variables

Examples

Wegner duality Wegner (1971)



Z_2 gauge theory $H = -\sum_{\text{link}} \sigma_1 - \sum_{\text{plaq}} \sigma_3\sigma_3\sigma_3\sigma_3$

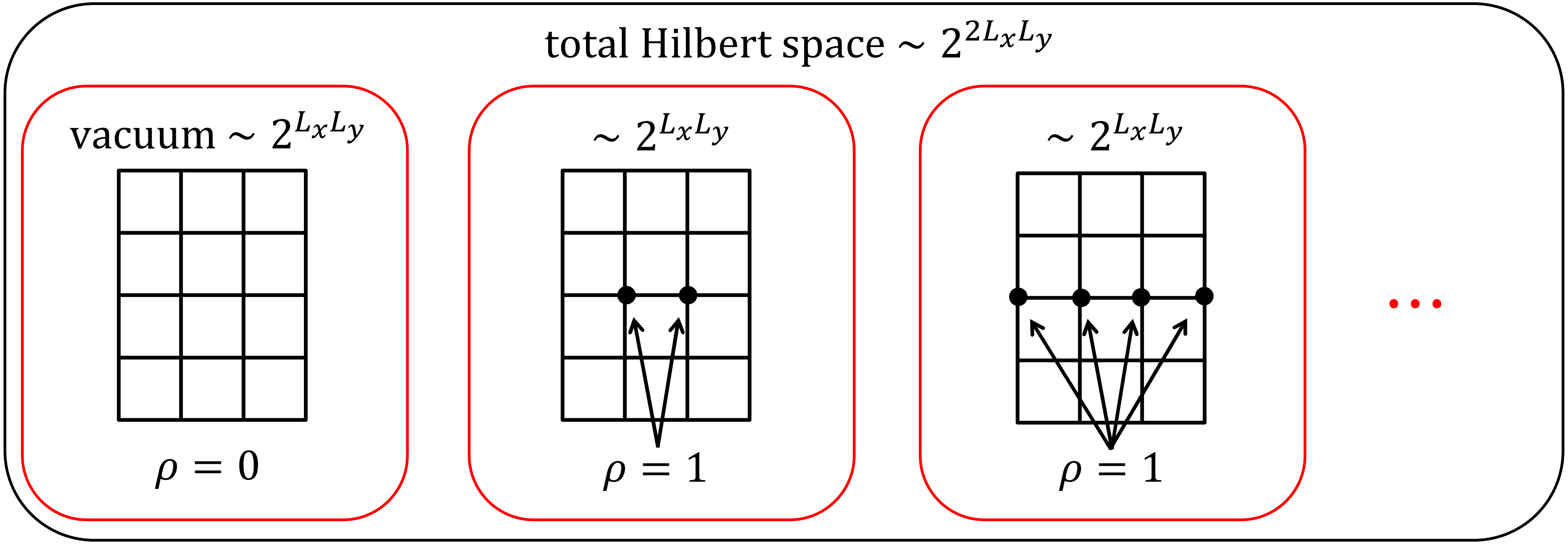
$\updownarrow$ duality transformation

Z_2 spin theory $H = -\sum_{\text{lin}} \sigma_3\sigma_3 - \sum_{\text{plaq}} \sigma_1$

Examples

Wegner duality

Wegner (1971)



total Hilbert space $\sim 2^{2L_xL_y}$

vacuum $\sim 2^{L_xL_y}$

$\rho = 0$

$\sim 2^{L_xL_y}$

$\rho = 1$

$\sim 2^{L_xL_y}$

$\rho = 1$

$\cdots$

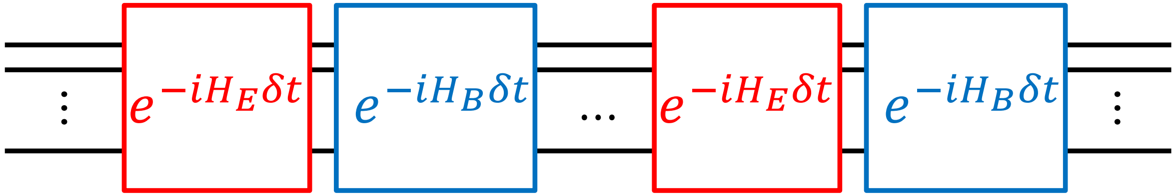
Examples

time evolution

initial state

$$|\Psi_0\rangle$$

quantum circuit



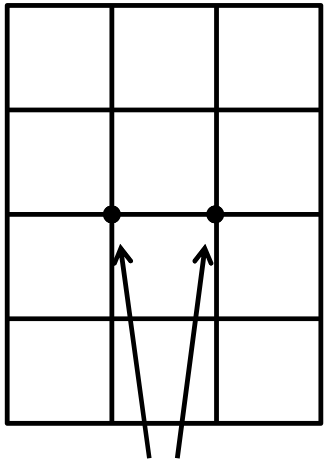
final state

$$|\Psi\rangle$$

Examples

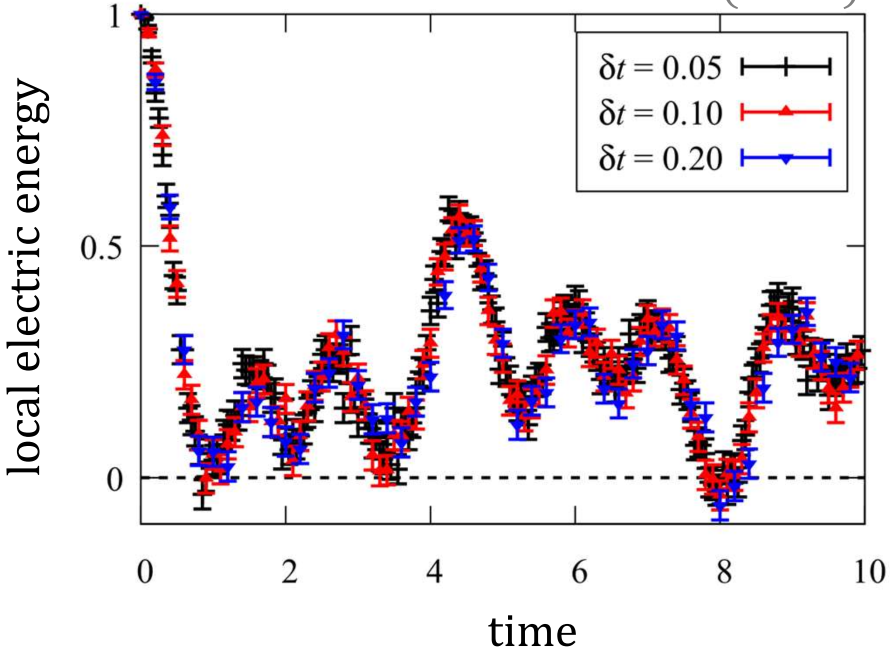
time evolution

two static charges



$\rho = 1$

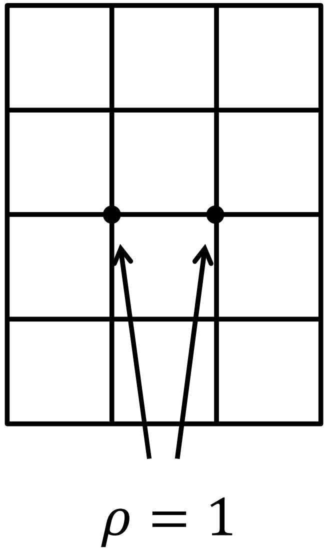
Yamamoto (2021)

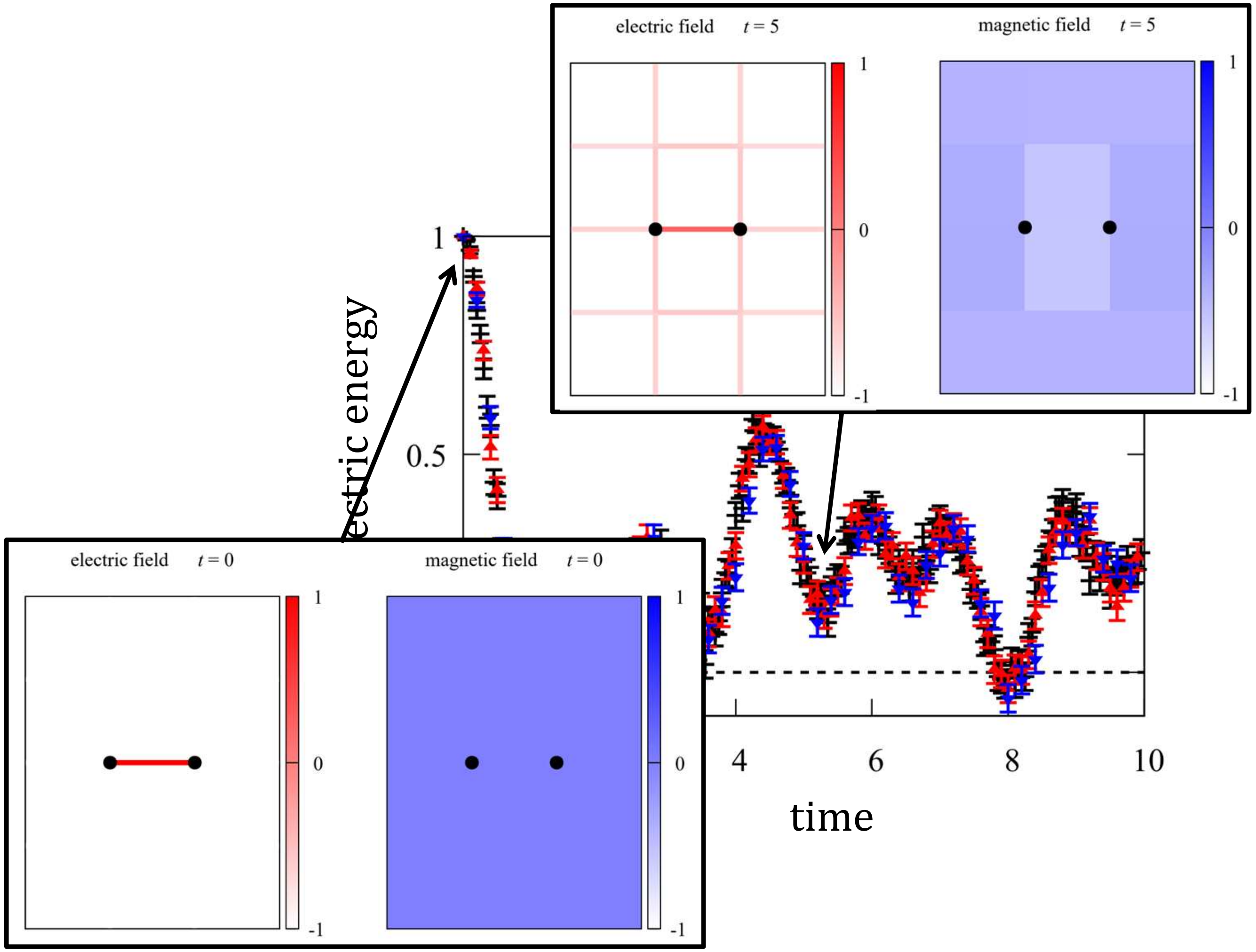


Examples

time evolution

two static charges

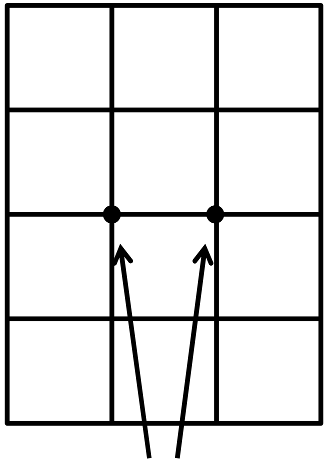




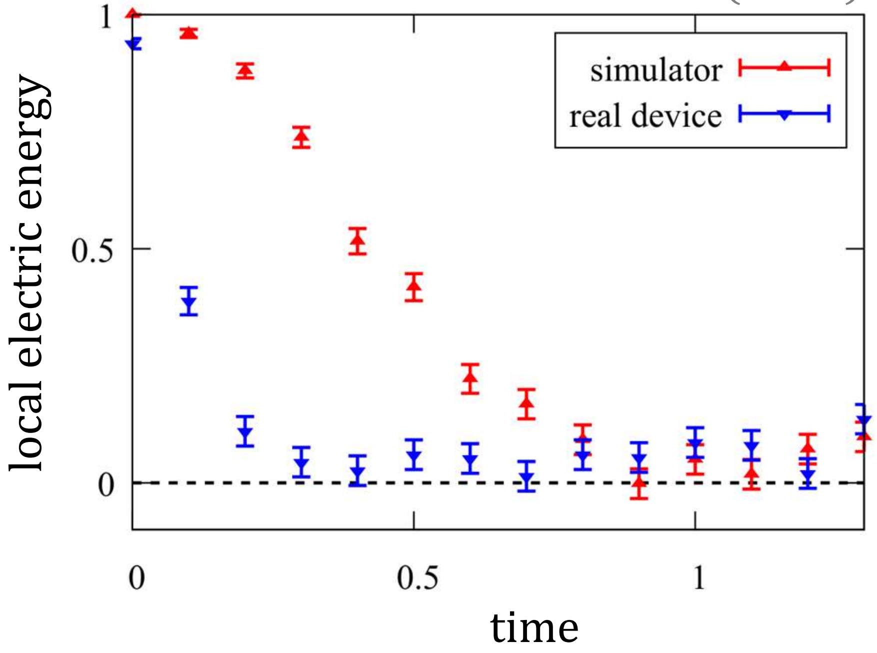
Examples

time evolution

two static charges



$\rho = 1$



Yamamoto (2021)

Examples

what to do:

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- ✓ continuum limit

specific problems in lattice gauge theory

(skipped in this talk)

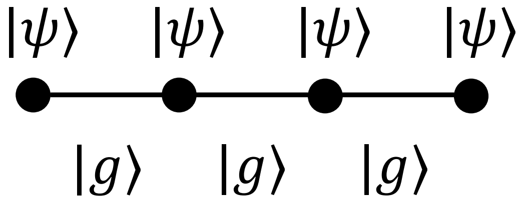
toy models:

- ✓ Z_n gauge theory
- ✓ 1-dim. gauge theory

explained by Sakamoto san & Hayata san

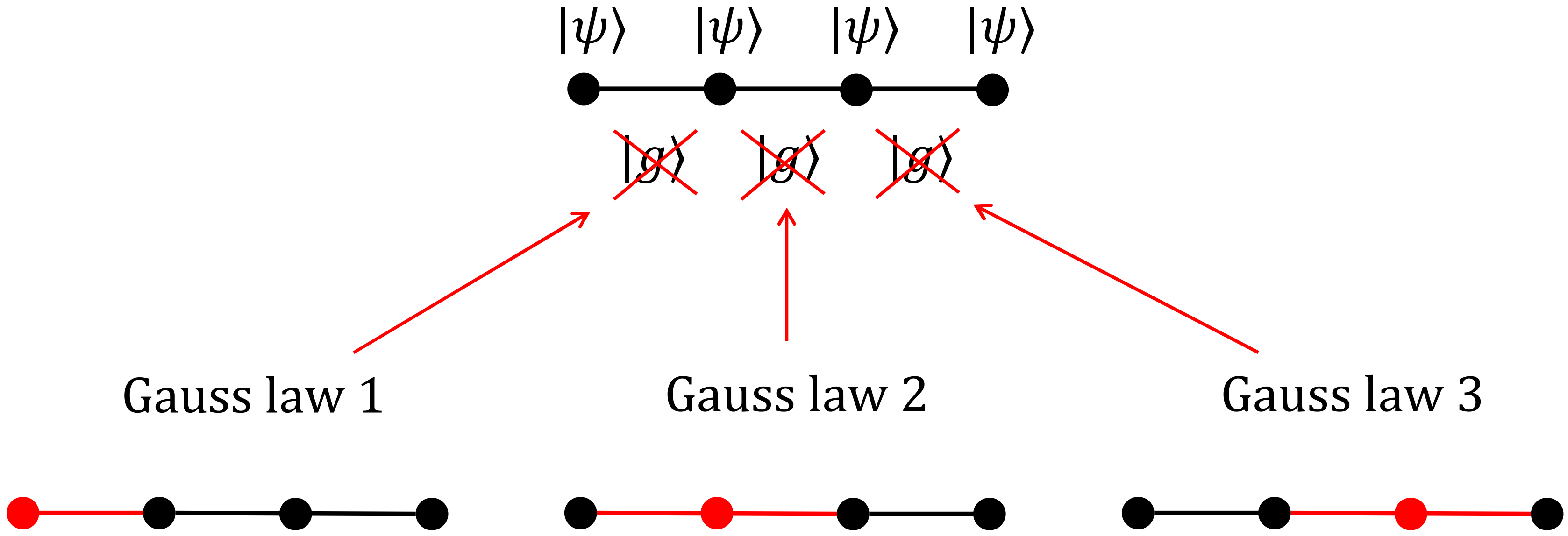
Examples

1D gauge + fermion theory (open boundary)



Examples

1D gauge + fermion theory (open boundary)



Gauss law 1

Gauss law 2

Gauss law 3

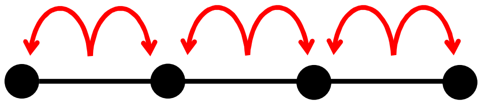
Examples

1. simulation w/ gauge fields

more qubits

$$|\Psi\rangle = \prod|\psi\rangle\prod|g\rangle$$

local gauge interaction

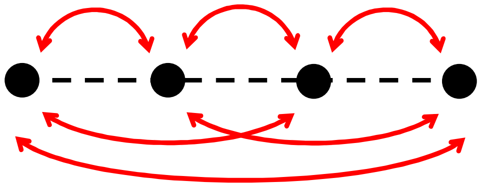


2. simulation w/o gauge fields

less qubits

$$|\Psi\rangle = \prod|\psi\rangle\xcancel{\prod|g\rangle}$$

non-local Coulomb interaction



Summary

✓ quantum simulation of lattice gauge theory has been outlined

✓ overlap with condensed matter physics & tensor network

✓ interdisciplinary study is welcome !!