テンソルネットワークによる 量子系の実時間シミュレーション

Simulating the real-time evolution of quantum systems by tensor networks

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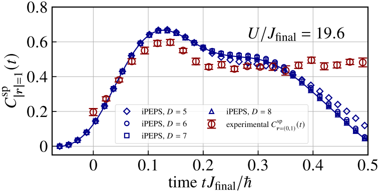
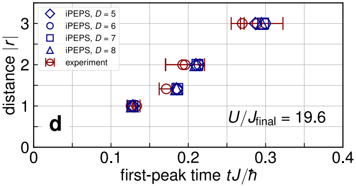
Collaborator: Ippei DANSHITA (Kindai Univ.)

R. Kaneko and I. Danshita, Commun. Phys. 5, 65 (2022)
R. Kaneko and I. Danshita, Phys. Rev. A 108, 023301 (2023)





Outline

- Introduction
  - Quench dynamics by analog quantum simulation
  - Importance of comparison with numerical simulations
  - Numerical difficulty in simulating the dynamics of 2D quantum systems

- Bose-Hubbard model: quench from a Mott insulating state
  - Motivation
    - Lack of reliable 2D methods
    - How far one can go by tensor-network states in 2D?
  - Tensor-network method
    - Simple update, projected entangled pair states (PEPS)
  - Results
    - Good agreement with experimental results
    - Estimate group and phase velocities for smaller U/J that has not been investigated in the experiment

- Transverse-field Ising model: quench from a disordered state
  - Motivation
    - To what extent is PEPS useful?
  - Preliminary results

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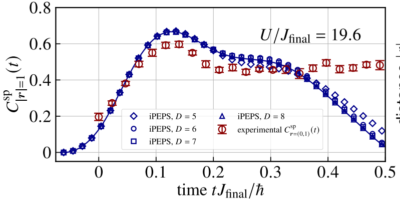

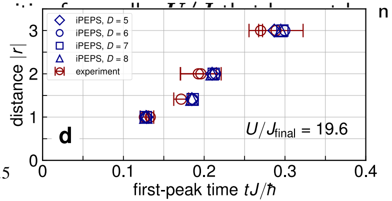

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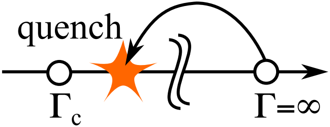

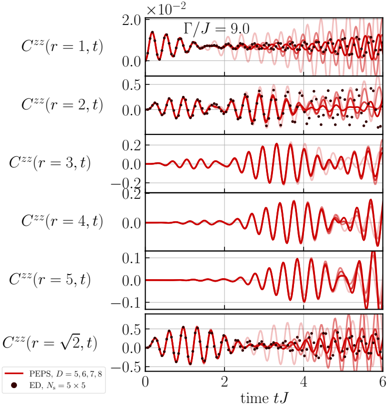

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  - Motivation
    - To what extent is PEPS useful?
  - Estimate group velocity and compare it with the current best estimate

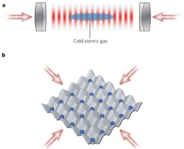
Introduction

Analog quantum simulators

Let the nature do the quantum simulations using highly controllable experimental devices

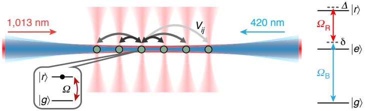
Ultracold atoms in optical lattices

[I.Bloch,Nature.453.1016('08); C.Gross,I.Bloch,Science.357.995('17); W.Hofstetter,T.Qin,J.Phys.B:At.Mol.Opt.Phys.51.082001('18)]



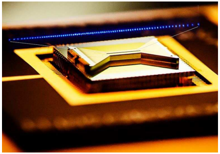
Rydberg atoms in optical tweezer arrays

[H.Bernien et al.,Nature.551.579('17); A.Keesling et al.,Nature.568.207('19)]



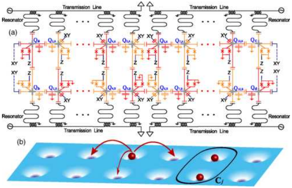
Trapped ion quantum computers

[R.Blatt,C.F.Roos,Nat.Phys.8.277('12); E.A.Martinez et al.,Nature.534.516('16); M.Gärtner et al.,Nat.Phys.13.781('17); `https://physicsworld.com/wp-content/uploads/2018/12/IonQ-chip.png`]



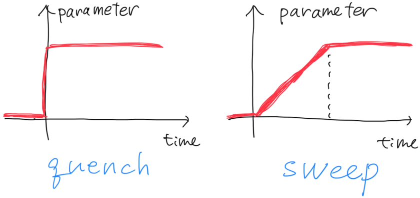
Superconducting quantum circuits

[R.Ma et al.,Nature.566.51('19); Y.Ye et al.,PRL.123.050502('19)]



What do we want to do using analog quantum simulators?

- Solve problems that are hard to tackle by classical computers
  - Prepare the Hamiltonian corresponding to the problem and obtain the equilibrium state (e.g. the ground state)
  - Simulate Schrödinger equation
    $\rightarrow$ Simulations of isolated quantum many-body systems have attracted much interest

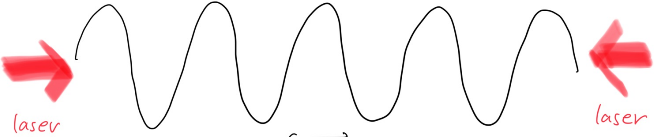


$*$ In experiments, quench is realized by very fast sweep

- In general, simulating time evolution requires all the information of eigenstates on classical computers
  $\rightarrow$ It is much harder than the ground-state calculation

In the case of ultracold atoms on optical lattices...

Standing wave generated by counterpropagating lasers → periodic potential = optical lattice



lattice spacing

potential

$V(x) = V_0 \sin^2(kx)$

V_0: intensity of laser

$d = \frac{\pi}{k} \sim 500\,\mathrm{nm}$ $\left(k = \frac{2\pi}{\lambda},\ \lambda : \text{wave length}\right)$

hopping J

onsite interaction U

Bose-Hubbard model : $\hat{H} = -J \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$

In the case of ultracold atoms on optical lattices...

$$V(x) = V_0 \sin^2(kx)$$

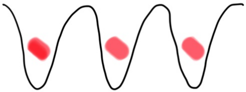
weaker V_0

U/J is small

$\Downarrow$

superfluid

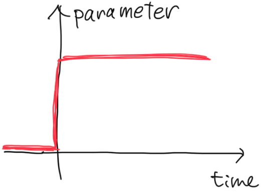
Stronger V_0



U/J is large

$\Downarrow$

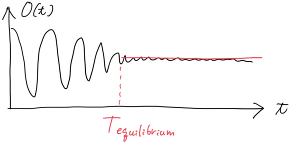
Mott insulator



quench

What do we want to clarify by simulating time evolution?

- How do isolated quantum many-body systems thermalize?



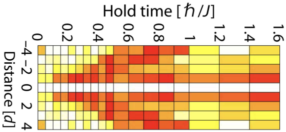
- What is the upper limit of the information propagation (= Lieb-Robinson bound)?

  cf. In relativistic system:
  Upper limit = speed of light

  Theoretical investigation is active recently

  cf. Light-cone-like behavior in Bose-Hubbard models

  [T.Kuwahara, K.Saito, PRL.127.070403('21)]



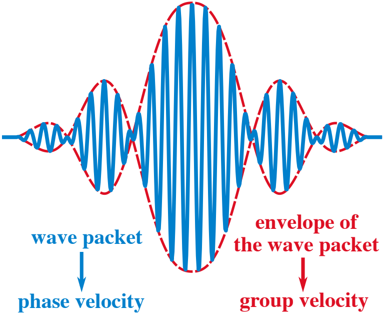
[Y.Takasu et al., sciadv.aba9255('20)]

Desirable to simulate the dynamics of correlation spreading to answer these questions

→ Longer-time experimental and numerical simulations are important

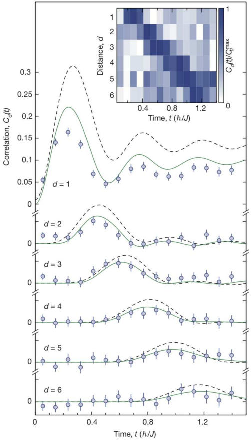
Propagation velocities can be obtained from equal-time correlations

- Two characteristic velocities
  - Phase velocity
  - Group velocity ($\leq$ Lieb-Robinson bound)



- In 1D, tensor-network simulations with matrix product states (MPS) are popular

- e.g. 1D Bose-Hubbard simulator
  Correlations after a quench
  [M.Cheneau et al.,Nature.481.484('11)]



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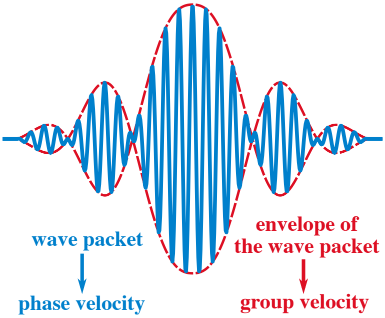

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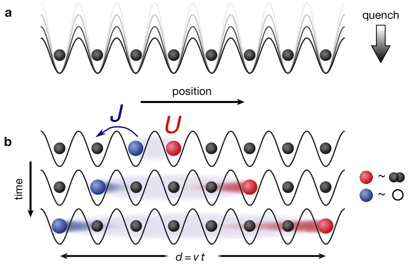
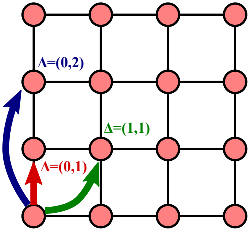
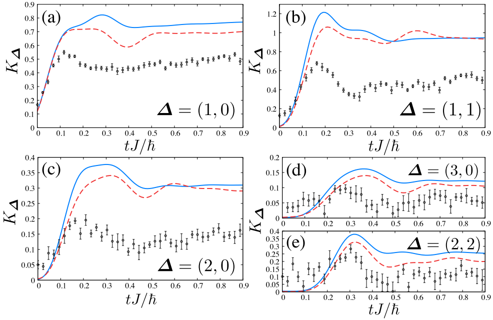
Quasiparticle picture



Dynamics of doublons and holons

Numerical simulations in 2D are extremely hard





[K.Nagao et al., PRR.3.043091('21)]

- e.g. Quench dynamics in the 2D Bose-Hubbard model
- Semiclassical approach (truncated Wigner approximation) is not powerful enough to reproduce the intensity of correlations
- Extend the 1D MPS wave functions to 2D
  Examine the accuracy of the 2D tensor-network method

Motivation

- Numerical simulations of time evolution on classical computers
- Crosscheck and predict experimental results
- Numerical simulations in 2D are extremely hard so far

- Focus on
  - 2D Bose-Hubbard model
  - 2D transverse-field Ising model

  to examine the accuracy of the 2D tensor-network method

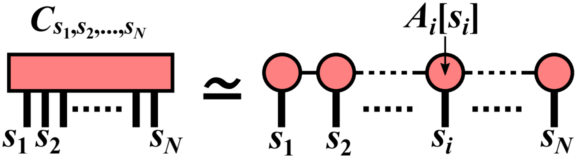
Tensor-network method

Tensor-network states: MPS and PEPS

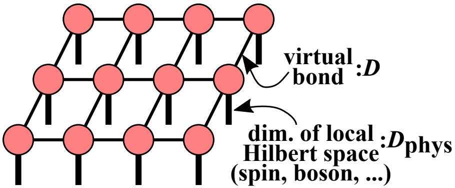
- Wave function for quantum spin systems:

$$|\psi\rangle = \sum_{\{s_i\}} C_{s_1,s_2,\ldots,s_N} |s_1, s_2, \ldots, s_N\rangle \qquad \#\text{elements} = O(e^N)$$

- In 1D: Matrix product state (MPS)



- In 2D: Projected entangled pair state (PEPS), tensor product state

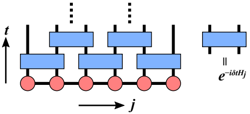


- $D_{\text{phys}} = 2S + 1$ for spin S
  (chosen to be sufficiently large for soft-core bosons)
- $D = 1$: direct product state
- $D \geq 2$: entangled state
- Wave functions are more accurate for larger D
- Translational invariant PEPS can treat infinite systems

[T.Nishino et al.,PTP.105.409('01); F.Verstraete, J.Cirac, arXiv:cond-mat/0407066]

Simulating real-time evolution by infinite PEPS

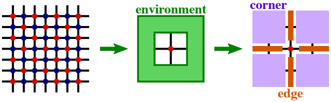
- Real-time evolution of infinite PEPS: $|\psi(t)\rangle = e^{-itH}|\psi(0)\rangle$



Time-evolving block decimation in 2D
(= simple update) [comp. cost: $O(D^5)$]

[H.C.Jiang,Z.Y.Weng,T.Xiang('08); P.Corboz et al.('10)]

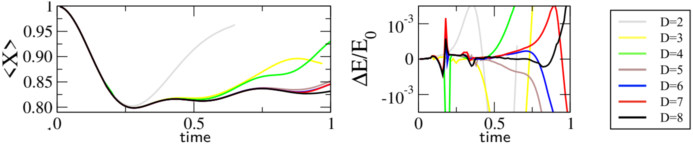
- Calculation of expectation values for infinite PEPS:



Corner transfer matrix renormalization group method [comp. cost $O(D^{10})$]

[R.J.Baxter('68); T.Nishino,K.Okunishi('96,'97); R.Orus,G.Vidal('09)]

- Previous studies on 2D quench dynamics (full update):
e.g. transverse-field Ising model (tr.-field: $h^x = \infty \to h^x_c$)
Time $\lesssim \hbar/J$ accessible by increasing bond dimension D



[A.Kshetrimayum et al.,Nat.Commun.8.1291('17); P.Czarnik et al.,PRB.99.035115('19); C.Hubig,J.I.Cirac,SciPost.Phys.6,031('19)]

Quench dynamics in the Bose-Hubbard model

Motivation:

- Reproduce experimental results
- Examine the parameter region that has not been explored

Numerical setup: Wish to calculate $|\psi(t)\rangle = e^{-i\hat{H}t}|\psi_0\rangle$

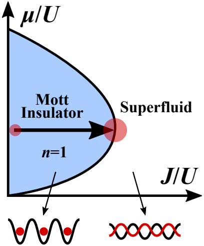
- Square Bose-Hubbard model: $\hat{H} = \sum_{\langle ij \rangle} \hat{H}_{ij}$

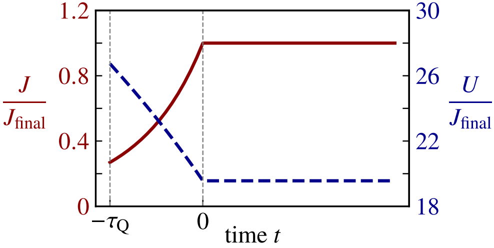
$$\hat{H}_{ij} = -J(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_i) + \frac{U}{2z}[\hat{n}_i(\hat{n}_i - 1) + \hat{n}_j(\hat{n}_j - 1)] - \frac{\mu}{z}(\hat{n}_i + \hat{n}_j) \quad (z = 4)$$

[V.Murg et al.,PRA('07); J.Jordan et al.,PRB('09); A.Kshetrimayum et al.,PRL('19); S.S.Jahromi and R.Orus,PRB('19); P.Schmoll et al.,PRL('20); W.-L.Tu et al.,JPCM('20); H.-K.Wu et al.,PRA('20); P.C.G.Vlaar and P.Corboz et al.,PRB('21)]

- Simple update by e.g. $e^{-idt\hat{H}/\hbar} \sim \prod_{\langle ij \rangle} e^{-idt\hat{H}_{ij}/\hbar}$

  (use second-order Suzuki-Trotter decomposition in practice)
- Very fast ($\tau_Q > 0$) and sudden ($\tau_Q = 0$) quenches from Mott insulator $\otimes_i|n_i = 1\rangle$
- Experimental setup: $U/J \sim 100 \to 19.6$ in $\tau_Q = 0.1$ms
  ($U/J = 19.6 > 16.74 = U_c/J$: Mott insulating region)



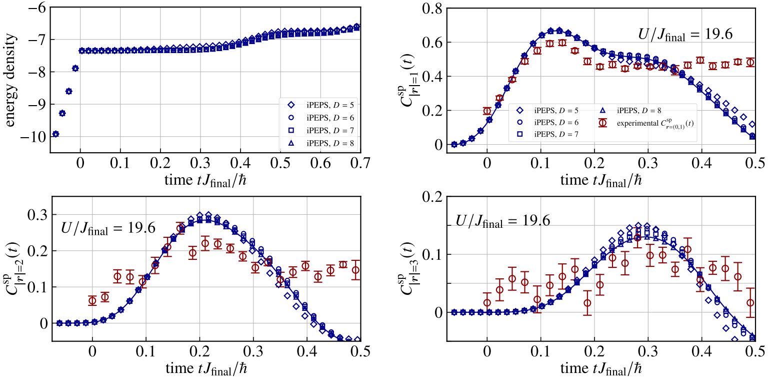


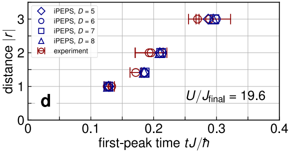
- Use tensor-network library TeNeS

[Y.Motoyama et al., Comp.Phys.Commun.279.108437('22); https://github.com/issp-center-dev/TeNeS, https://github.com/TsuyoshiOkubo/pTNS]

Numerical results: Comparison with the experiment at $U/J = 19.6$

$$C_r^{\mathrm{sp}}(t) = \frac{1}{2N_{\mathrm{s}}} \sum_{r_i - r_j = r} \langle \hat{a}_i^\dagger(t)\hat{a}_j(t) + \hat{a}_j^\dagger(t)\hat{a}_i(t) \rangle$$

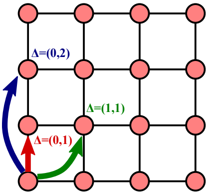


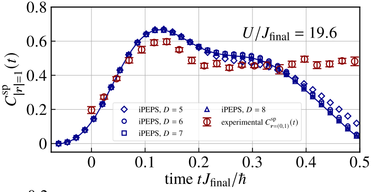


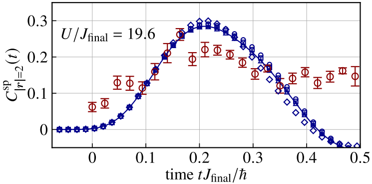
- Consider finite quench time as in the experiment
- Nearly conserved energy for $0 \leq tJ/\hbar \lesssim 0.4$
- Physical quantities are likely to be converged for this short time

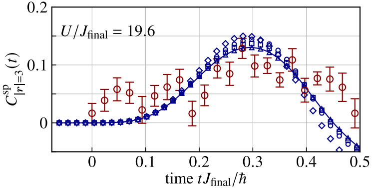
Numerical results: Comparison with the experiment at $U/J = 19.6$

$$C_r^{\mathrm{sp}}(t) = \frac{1}{2N_{\mathrm{s}}} \sum_{r_i - r_j = r} \langle \hat{a}_i^\dagger(t)\hat{a}_j(t) + \hat{a}_j^\dagger(t)\hat{a}_i(t)\rangle$$

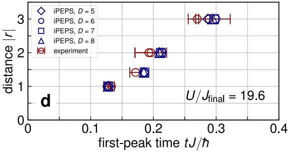




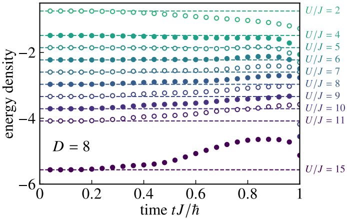


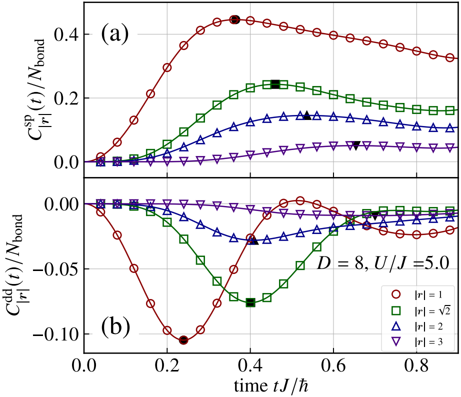


- Consider finite quench time as in the experiment
- Nearly conserved energy for $0 \leq tJ/\hbar \lesssim 0.4$
- Single-particle correlations agree very well
- How about other parameter regions?
- How does the propagation velocity behave?
- Set quench time $\tau_Q = 0$ hereafter



Numerical results: Estimate propagation velocities from $\langle a_0^\dagger a_r \rangle$ and $\langle n_0 n_r \rangle$





Consider a sudden quench

Energy is conserved for longer time $tJ/\hbar \lesssim 0.9$ when $U/J \sim 5$

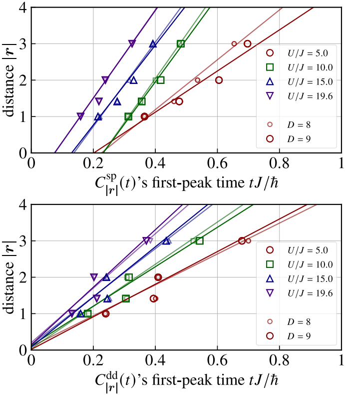
Can capture peaks up to $|r| \leq 3$

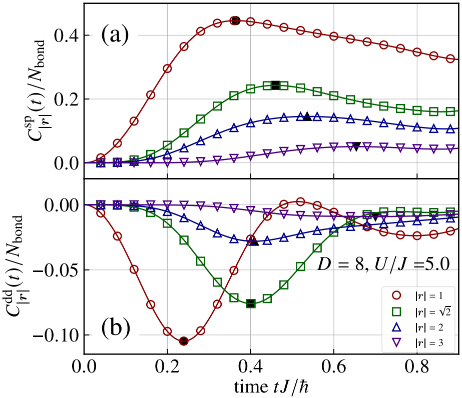
$$C_r^{\mathrm{sp}}(t) = \frac{1}{2N_{\mathrm{s}}} \sum_{r_i - r_j = r} \langle \hat{a}_i^\dagger(t)\hat{a}_j(t) + \hat{a}_j^\dagger(t)\hat{a}_i(t) \rangle$$

$$C_r^{\mathrm{dd}}(t) = \frac{1}{N_{\mathrm{s}}} \sum_{r_i - r_j = r} (\langle \hat{n}_i(t)\hat{n}_j(t) \rangle - 1)$$

- v_{phase}: captured by single-particle correlation $\langle a_0^\dagger a_r \rangle$
- v_{group}: captured by density-density correlation $\langle n_0 n_r \rangle$

Numerical results: Estimate propagation velocities from $\langle a_0^\dagger a_r \rangle$ and $\langle n_0 n_r \rangle$



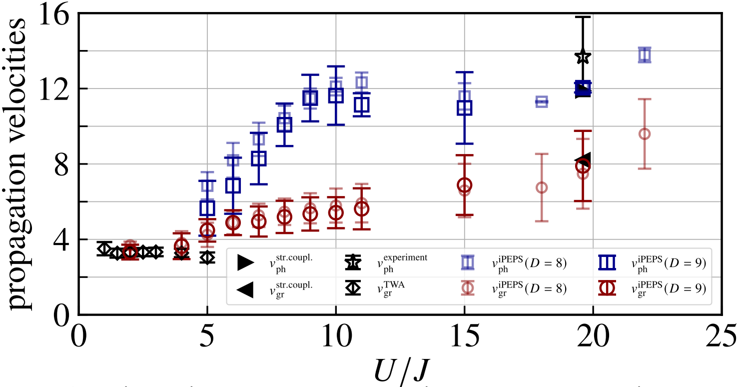


$$C_r^{\rm sp}(t) = \frac{1}{2N_{\rm s}} \sum_{r_i - r_j = r} \langle \hat{a}_i^\dagger(t)\hat{a}_j(t) + \hat{a}_j^\dagger(t)\hat{a}_i(t) \rangle$$

$$C_r^{\rm dd}(t) = \frac{1}{N_{\rm s}} \sum_{r_i - r_j = r} \left(\langle \hat{n}_i(t)\hat{n}_j(t) \rangle - 1 \right)$$

- $v_{\rm phase}$: captured by single-particle correlation $\langle a_0^\dagger a_r \rangle$
- $v_{\rm group}$: captured by density-density correlation $\langle n_0 n_r \rangle$

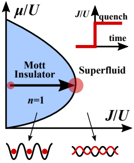
Numerical results: U dependence of velocity

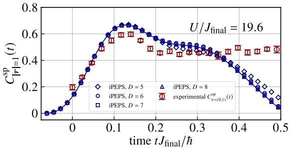


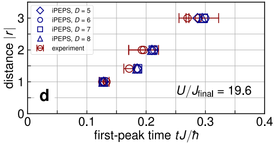
- For $U \lesssim zJ$ $(z=4)$, single-particle picture (mean-field-like picture) holds
  $v_{\mathrm{group}} \sim 4J/\hbar$ [K.Nagao et al.,PRA.99.023622('19)]
- For $U \gg J$, quasi-particle picture holds
  $v_{\mathrm{group}} \sim 6J/\hbar \times [1+\mathcal{O}(J^2/U^2)]$ [M.Cheneau et al.,Nature.481.484('11)]
- v_{group} estimated from $\langle n_0 n_r \rangle$ consistent with
  - single-particle group velocity deep in superfluid region
  - strong-coupling result near criticality
- v_{phase} and v_{group} gradually converge to the same value as U/J is decreased

Conclusions: Bose-Hubbard case

- Quench dynamics from Mott insulator in 2D Bose-Hubbard model
- Simulation by infinite PEPS using simple update
- Compare PEPS simulations with experiments
  $\rightarrow$ Good agreement for $tJ/\hbar \lesssim 0.4$ at $U/J = 19.6$



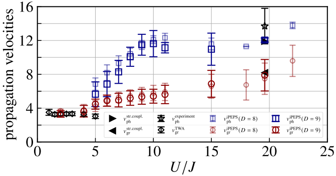




- Estimate velocity of correlation spreading for smaller U/J

v_{phase} and v_{group} gradually converge to the same value as U/J is decreased

Might be helpful for future experiments



R. Kaneko and I. Danshita, Commun. Phys. 5, 65 (2022)

Quench dynamics in the 2D transverse-field Ising model

Motivation:

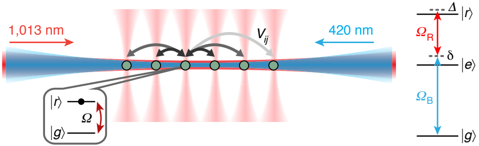
- How good is the 2D tensor-network method in this case?
- How does the group velocity for spin correlations look?
  (Compare with the recently updated Lieb-Robinson bound)

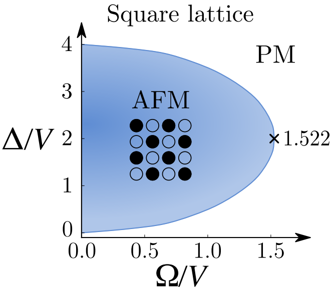
Analog quantum simulations of the quantum Ising model by Rydberg-atom arrays

[H.Bernien et al.,Nat.551.579('17); A.Keesling et al.,Nat.568.207('19); E.Guardado-Sanchez et al.,PRX.8.021069('18); V.Lienhard et al.,PRX.8.021070('18); D.Bluvstein et al.,Science.371.1355('21); P.Scholl et al.,Nat.595.233('21); S.Ebadi et al.,Nat.595.227('21); ...]

$$H = \Omega \sum_i S_i^x - \Delta \sum_i n_i + V \sum_{\langle ij \rangle} n_i n_j \quad \left(n_i = S_i^z + \frac{1}{2},\ D:\ \text{dim.} \right)$$

$$= \Omega \sum_i S_i^x - (\Delta - VD) \sum_i S_i^z + V \sum_{\langle ij \rangle} S_i^z S_j^z + \sum_i \left(\frac{VD}{4} - \frac{\Delta}{2} \right)$$





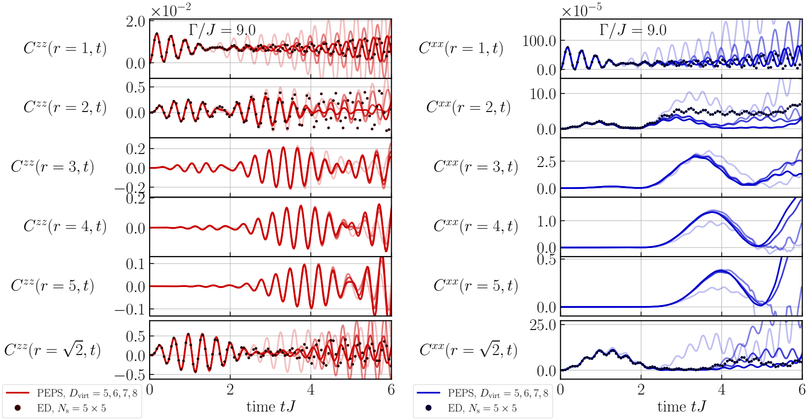
	Rydberg atoms	quantum Ising model
bases	$\|g\rangle, \|r\rangle$	$\|\downarrow\rangle, \|\uparrow\rangle$
Ω	Rabi frequency	transverse field
Δ	detuning	longitudinal field
V	van der Waals interaction	Ising spin interaction

Very recently, real-time dynamics for # qubits > 200

How does information propagate?

Numerical results: 2D, iPEPS

$$C^{zz}(r,t) = \langle\psi(t)|\hat{S}^z_r \hat{S}^z_0|\psi(t)\rangle$$
$$C^{xx}(r,t) = \langle\psi(t)|\hat{S}^x_r \hat{S}^x_0|\psi(t)\rangle - \langle\psi(t)|\hat{S}^x_r|\psi(t)\rangle\langle\psi(t)|\hat{S}^x_0|\psi(t)\rangle$$



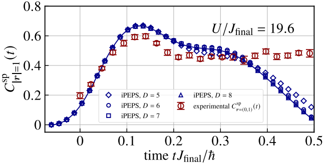
Distances longer than those by the exact diagonalization method are calculable

Conclusions

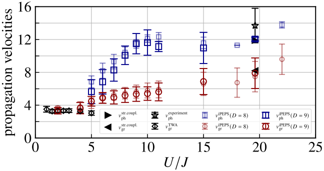
- Simulating the dynamics of 2D systems by the tensor-network method with iPEPS
- Focus on the quench in the 2D Bose-Hubbard and transverse-field Ising models

<u>Bose-Hubbard case</u>

- Good agreement with the experiment

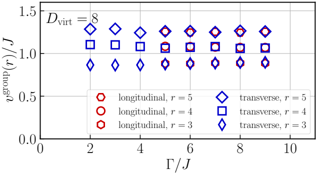


- Examine the parameter region that has not been explored



<u>Ising case</u>

- Group velocity satisfies the Lieb-Robinson bound (but the value is much smaller than the bound)



- $\boldsymbol{v_{\mathrm{spin}}/J \approx 1}$ both in 1D and 2D
- Recent estimate $\boldsymbol{v_{\mathrm{LR}}/J = 5.672}$ is still loose?

- Provide numerical data that can be compared with future experiments

R. Kaneko and I. Danshita, Commun. Phys. 5, 65 (2022)

R. Kaneko and I. Danshita, Phys. Rev. A 108, 023301 (2023)

Backup slides

Quench dynamics in the 2D transverse-field Ising model

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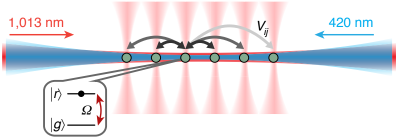
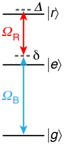
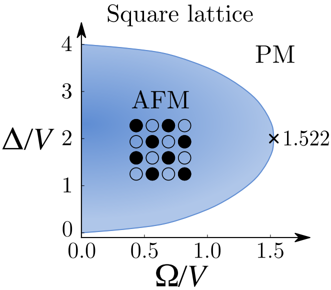
$$\begin{aligned} H &= \Omega \sum_i S_i^x - \Delta \sum_i n_i + V \sum_{\langle ij \rangle} n_i n_j \quad \left(n_i = S_i^z + \frac{1}{2},\ D:\ \text{dim.} \right) \\ &= \Omega \sum_i S_i^x - (\Delta - VD) \sum_i S_i^z + V \sum_{\langle ij \rangle} S_i^z S_j^z + \sum_i \left(\frac{VD}{4} - \frac{\Delta}{2} \right) \end{aligned}$$







	Rydberg atoms	quantum Ising model
bases	$\lvert g\rangle, \lvert r\rangle$	$\lvert\downarrow\rangle, \lvert\uparrow\rangle$
$\boldsymbol{\Omega}$	Rabi frequency	transverse field
$\boldsymbol{\Delta}$	detuning	longitudinal field
$\boldsymbol{V}$	van der Waals interaction	Ising spin interaction

Very recently, real-time dynamics for # qubits $>$ **200**

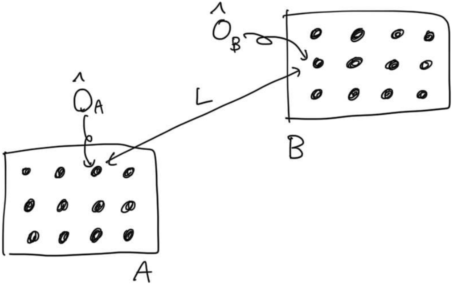
How does information propagate?

Lieb-Robinson bound: Upper limit of group velocity for any correlations

- Consider regions A and B in a lattice system with short-range interaction
- Commutator of any operators O_A and O_B in regions A and B satisfies

$$||[O_A(t), O_B]|| \leq \text{const.} \times \exp\left(-\frac{L - vt}{\xi}\right)$$

- $O_A(t) = e^{iHt} O_A e^{-iHt}$, L: distance between A and B, ξ: const.
- v: Lieb-Robinson bound

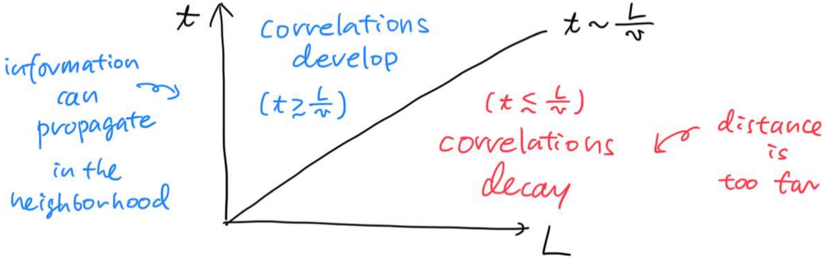


- Information from A is transmitted to B up to $t \sim L/v$
- It tells the presence of bound, but not the value itself

Light-cone-like spreading of time-dependent correlations

- Corollary: For a state with finite correlation length ξ
  - Bound for any equal-time correlations:
    [S.Bravyi et al.,PRL.97.050401('06)]

$$|\langle O_A(t) O_B(t)\rangle - \langle O_A(t)\rangle\langle O_B(t)\rangle| \leq \text{const.} \times e^{-\frac{L-2vt}{\xi}}$$

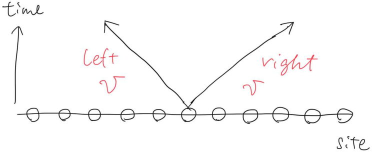


- Velocity v: Lieb-Robinson bound
- Exact Lieb-Robinson bound is known only for a few lattice models (e.g. 1D systems)
- Lieb-Robinson bound gets tighter very recently
  [Z.Wang,K.R.A.Hazzard,PRXQuant.1.010303('20)]

How to get/estimate Lieb-Robinson velocity

- Bound for any equal-time correlations: [S.Bravyi et al.,PRL.97.050401('06)]

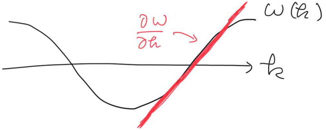
$$|\langle O_A(t) O_B(t)\rangle - \langle O_A(t)\rangle\langle O_B(t)\rangle| \le \text{const.} \times e^{-\frac{L-2vt}{\xi}}$$



  - Intuitively, prefactor $\mathbf{2}$ comes from left and right moving quasiparticles

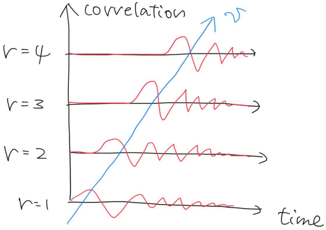
From dispersion

- $v = \max_k \dfrac{d\omega(k)}{dk}$
- In 1D TFIsing, $\omega(k)$ is exactly known and the exact v is obtained
- Precise shape of dispersion $\omega(k)$ is not known in general



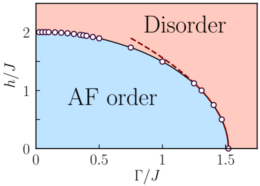
From peak localtions in correlations

- $v_{\text{experiment}} = 2v$
- Useful in experiments



Numerical setup: Transverse-field Ising model

- Ground-state phase diagram in 2D [R.Kaneko et al.,JPSJ.90.073001('21)]

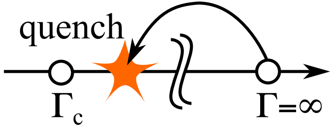


$$\boldsymbol{H} = +\boldsymbol{J}\sum_{\langle ij\rangle}\boldsymbol{S_i^z S_j^z} - \boldsymbol{\Gamma}\sum_{i}\boldsymbol{S_i^x} - \boldsymbol{h}\sum_{i}\boldsymbol{S_i^z}$$

- For simplicity, focus on $\boldsymbol{h = 0}$ case
  $\rightarrow$ Map to ferromagnetic model by appropriate unitary transformation

$$\boldsymbol{H} = -\boldsymbol{J}\sum_{\langle ij\rangle}\boldsymbol{S_i^z S_j^z} - \boldsymbol{\Gamma}\sum_{i}\boldsymbol{S_i^x}$$

- Sudden quench from the $\boldsymbol{\Gamma = \infty}$ ground state $|\rightarrow\rightarrow\cdots\rightarrow\rangle$



$$|\psi(t)\rangle = e^{-iHt/\hbar}|\rightarrow\rightarrow\cdots\rightarrow\rangle$$

$$\boldsymbol{\Gamma_{c,1D}/J = 0.5}$$
$$\boldsymbol{\Gamma_{c,2D}/J \approx 1.522}$$

- How does the group velocity for spin correlations look?

Numerical setup: Considered dimensions and methods

Estimate $\boldsymbol{v}_{\textbf{experiment}} = \boldsymbol{2v}$ numerically from peak locations
For comparison, we also consider the 1D case

	exact	tensor network	spin wave approx.
1D	✓	✓(MPS*)	✓
2D	×	✓(iPEPS)	✓

$*$ The data is not shown because it was consistent with the exact result

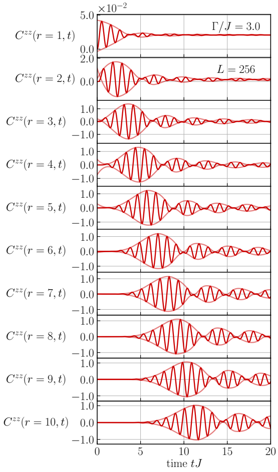
- 1D, exact
  - Jordan-Wigner transformation: Spin $\rightarrow$ Fermion
  - Time-dependent correlations: Pfaffian ($= \pm\sqrt{\text{determinant}}$) of the matrix containing two-body correlations of fermions

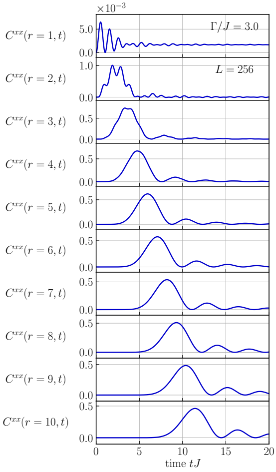
- 1D, 2D, spin wave approx.
  - Holstein-Primakoff transformation: Spin $\rightarrow$ Boson (magnon)
  - Time-dependent correlations: Function of magnon dispersions

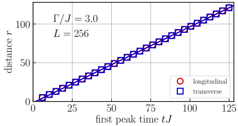
Numerical results: 1D, exact

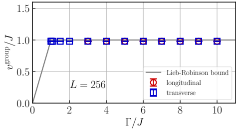
$$C^{zz}(r,t) = \langle\psi(t)|\hat{S}^z_r\hat{S}^z_0|\psi(t)\rangle$$

$$C^{xx}(r,t) = \langle\psi(t)|\hat{S}^x_r\hat{S}^x_0|\psi(t)\rangle - \langle\psi(t)|\hat{S}^x_r|\psi(t)\rangle\langle\psi(t)|\hat{S}^x_0|\psi(t)\rangle$$





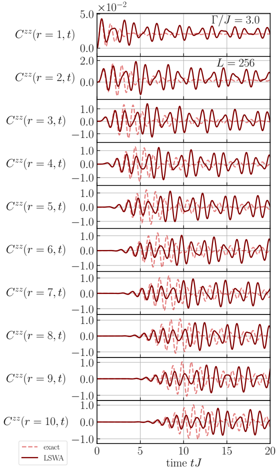


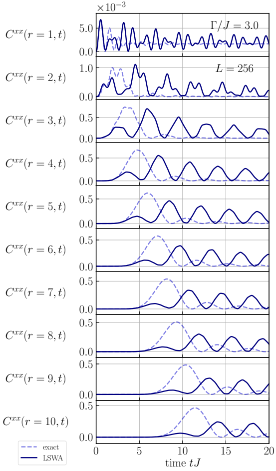


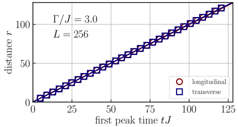
- Light-cone-like spreading of correlations
- Group velocity of spin-spin correlations = Lieb-Robinson velocity ($v/J = 1$)
- Quasiparticles with the fastest propagation velocity are responsible for the spreading of spin-spin correlations

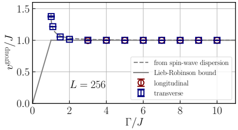
Numerical results: 1D, approximate method
Focus on the spin wave approx.

$$C^{zz}(r,t) = \langle \psi(t)|\hat{S}^z_r \hat{S}^z_0|\psi(t)\rangle$$
$$C^{xx}(r,t) = \langle \psi(t)|\hat{S}^x_r \hat{S}^x_0|\psi(t)\rangle - \langle \psi(t)|\hat{S}^x_r|\psi(t)\rangle\langle \psi(t)|\hat{S}^x_0|\psi(t)\rangle$$





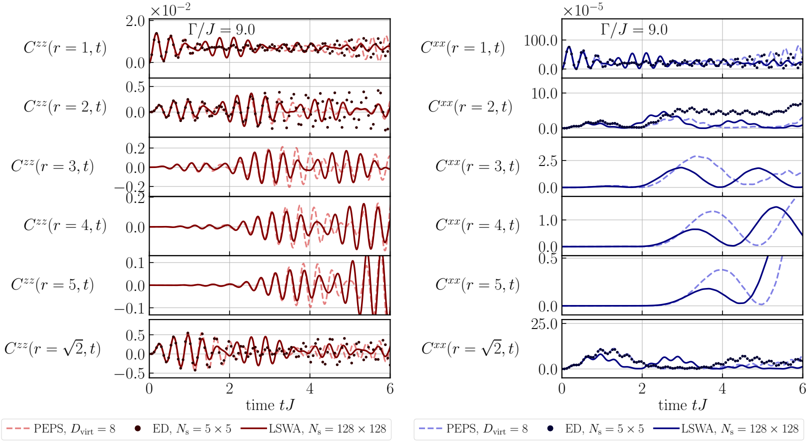




- Accurate up to the point where they begin to increase
- Group velocity of spin-spin correlations $\longrightarrow$ Lieb-Robinson velocity ($v/J = 1$) when $\Gamma \gg J$
- For a small quench ($\Gamma = \infty$ to $\Gamma \gg J$), few quasiparticles are involved, and SW approx. is good

Numerical results: 2D, spin wave approx.
(should be good for higher dimensions)

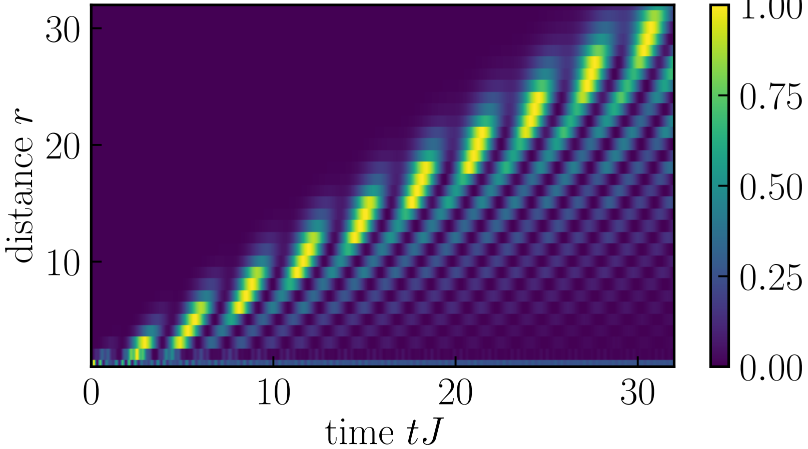
$$C^{zz}(r,t) = \langle\psi(t)|\hat{S}^z_r \hat{S}^z_0|\psi(t)\rangle$$
$$C^{xx}(r,t) = \langle\psi(t)|\hat{S}^x_r \hat{S}^x_0|\psi(t)\rangle - \langle\psi(t)|\hat{S}^x_r|\psi(t)\rangle\langle\psi(t)|\hat{S}^x_0|\psi(t)\rangle$$



Agreement is slightly better than in 1D

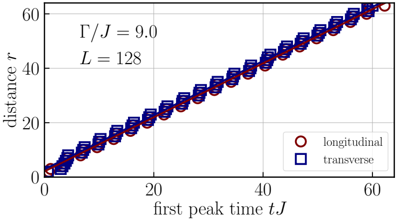
Numerical results: 2D, spin wave approx.

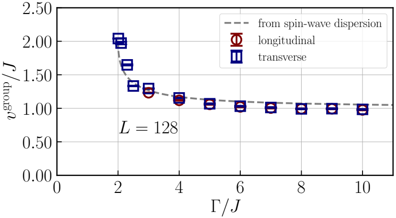
normalized correlation function
$\boldsymbol{C^{xx}(r,t)/\max_{t\in[0,L/(2J)]} C^{xx}(r,t)}$ ($\in \boldsymbol{[0,1]}$)



Areas of high intensity are not continuously connected
(Complex interference effects in 2D?)

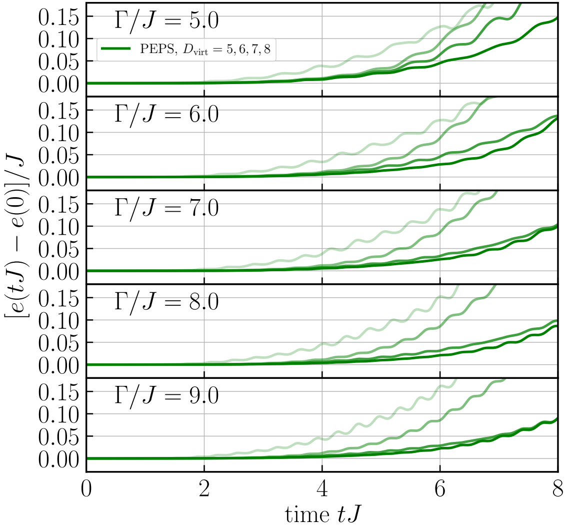
Numerical results: 2D spin wave approx.





- Light-cone-like spreading of correlations in 2D as well
- For $\boldsymbol{\Gamma/J \gg 1}$, the group velocity estimated from the peak location is $\boldsymbol{v/J \approx 1}$
- The group velocity estimated from the SW dispersion $\boldsymbol{\Omega_k = \sqrt{\Gamma^2 - \frac{z}{2}\Gamma J \gamma_k}}$ ($\boldsymbol{\gamma_k = \frac{1}{D}\sum_{\nu=1}^{D} \cos k_\nu}$, $\boldsymbol{z = 2D}$, $\boldsymbol{D}$: dimension) is $\boldsymbol{v^{\text{SW}}/J = (1 - J/\Gamma + \sqrt{1 - 2J/\Gamma})^{-1/2}/\sqrt{2}}$, which also approaches $\boldsymbol{v/J \approx 1}$ for $\boldsymbol{\Gamma/J \gg 1}$

Numerical results: 2D, iPEPS



The energy is nearly conserved for $\boldsymbol{D_{\mathrm{virt}} \geq 6}$
in a time frame $\boldsymbol{tJ \in [0, 4]}$

Numerical results: 2D, iPEPS

$$C^{zz}(r,t) = \langle \psi(t) | \hat{S}^z_r \hat{S}^z_0 | \psi(t) \rangle$$

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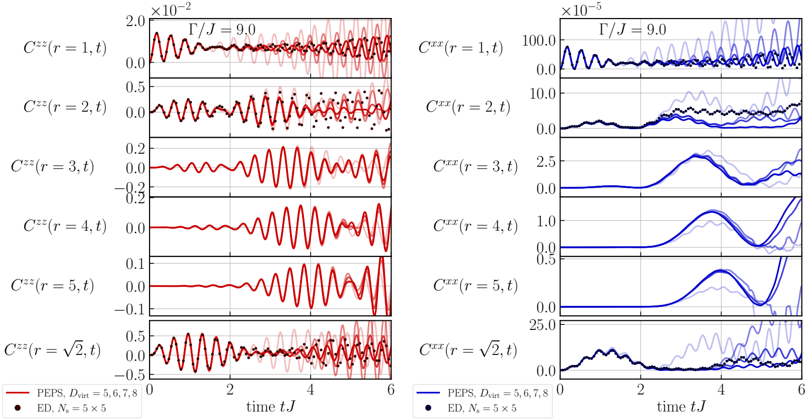
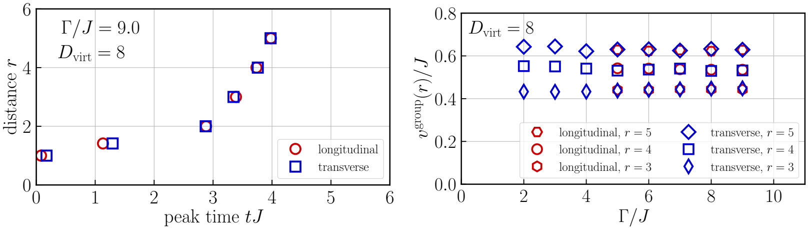


Distances longer than those by the exact diagonalization method are calculable

Numerical results: 2D, iPEPS



- Since the energy is conserved for a short time, accessible time is limited
- Not easy to esitimate the group velocity from iPEPS data
- Assume light-cone-like spreading of correlations (as in SW approx.) exists and the peak localtion eventually grows linearly with the peak time
- Estimate $\boldsymbol{v = r/t(r)}$ for each distance $\boldsymbol{r}$: $\boldsymbol{v/J \in [0.86, 1.3]}$

Comparison of group velocities v/J at $\Gamma/J \gg 1$

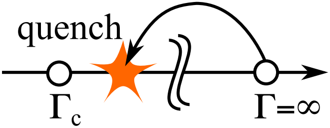
	from dispersion (fermion or magnon)	from peak location $(C^{zz,xx}(r))$	from recent inequality* (any correlations)
1D, exact	1 (exact)	$\mathbf{1.0}$	$\leq \mathbf{3.02}$
1D, SW	$\mathbf{1.0}$	$\mathbf{1.0}$	
2D, PEPS	N/A	$\in [\mathbf{0.86, 1.3}]$	$\leq \mathbf{5.672}$
2D, SW	$\mathbf{1.0}$	$\mathbf{1.0}$	

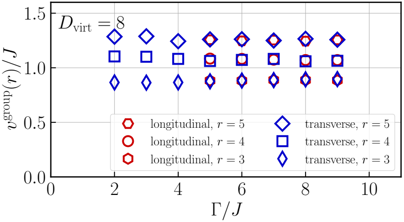
$*$ [Z.Wang,K.R.A.Hazzard,PRXQuant.1.010303('20)]

- In 1D, the spin correlation provides the group velocity corresponding to the fastest quasiparticle (identical to exact Lieb-Robinson bound)
- If this is so in 2D as well, LR bound in 2D TFIsing would also be $\boldsymbol{v/J = 1} \rightarrow$ Room for improvement in LR bound?

Conclusions: Ising case

- Quench dynamics from the disordered state in 2D transverse-field Ising model
- Simulation by infinite PEPS using simple update





- Our estimate of the group velocity: $\boldsymbol{v_{\mathrm{spin}}/J \sim 1}$
- This is much smaller than the current best Lieb-Robinson bound: $\boldsymbol{v_{\mathrm{LR,horiz}}/J = 5.672}$
- Our group velocity and spin correlations are helpful for crosschecking experimental data

R. Kaneko and I. Danshita, Phys. Rev. A 108, 023301 (2023)